Name:_____

Math 402, Section C13 - Final Exam - May 5, 2006

Time: 3 hours. Write your answers on the blank paper provided. Be sure to number the problems. You may not use any books or notes (except for those provided) or calculator. There are 200 points possible. To get full credit, you must show your work.

1. (5 points each) Give the definition of each of the following:
   (a) *congruent triangles.*
   (b) *Euclidean isometry.*
   (c) *Omega point.*
   (d) *Lambert quadrilateral.*
   (e) the *defect* of a triangle in hyperbolic space.

2. (30 points) Consider the following axiomatic system. Answer the questions (a)-(c), giving a proof for each one, or at least the strongest evidence that you can.

   A1 There are at least four students.

   A2 Any two students are in exactly one class together.

   A3 Each class has at least two students in it.

   (a) Is this axiomatic system consistent?
   (b) Is A3 an independent axiom in this system?
   (c) Is this axiomatic system complete?

3. (20 points) Using the SAS congruence theorem, prove the ASA congruence theorem. (Hint: try a proof by contradiction)

4. (20 points) Prove that an isometry maps parallel lines to parallel lines. (You may assume that an isometry maps lines to lines).

5. (20 points) List the 4 kinds of Euclidean isometries. For each one, tell what its fixed points are and whether it preserves orientation or reverses orientation.

6. (20 points) Prove that two Saccheri quadrilaterals with congruent summits and congruent summit angles must be congruent quadrilaterals; that is, the bases must be congruent and the sides must be congruent.

7. (20 points) For triangles inn hyperbolic geometry, what is the relationship between defect and area? Look at the two triangles drawn in the Poincare model on the page provided. Which has the larger area? Explain your answer, making use of the idea of defect.

8. (24 points) List eight differences between Euclidean geometry and hyperbolic geometry.

9. (3 points each part) Answer true or false (no explanation needed and no partial credit on this problem)

   (a) It is possible to tile the hyperbolic plane with 45, 45, 45 triangles (i.e. the three angles in each triangle are all 45 degrees).

   (b) In hyperbolic space, two triangles having the same area must be congruent.

   (c) A pentagon in hyperbolic space must have angle sum less than 540 degrees.

   (d) In hyperbolic space, if line m is a right-limiting parallel to line l, then l is conversely a right-limiting parallel to m.

   (e) In the Poincare model, distance is measured by taking the Euclidean arc length along a hyperbolic line segment.

   (f) Every Euclidean isometry can be written as the composition of three or fewer reflections.

   (g) Pasch's axiom was introduced to deal with certain hidden assumptions in Euclid's proofs.