

Name: _____

Math 402, Section C13 - Test #1 - February 20, 2006

Time: 55 minutes. Write your answers on the blank paper provided. Start a new page for each problem and be sure to number the problems. You may not use any books or notes (except for those provided) or calculator. There are 100 points possible. To get full credit, you must show your work.

1. (20 points) State Euclid's five Postulates (axioms). Also explain briefly which one of these postulates is false for hyperbolic space.

2. (20 points) Suppose a very small and very intelligent bug is living in either Euclidean space, hyperbolic space, or spherical space, but he does not know which one. This bug has the ability to walk in a straight line (as defined in Euclidean, hyperbolic or spherical space), to walk in a circle, to trace his path with chalk as he goes, and to measure angles. Describe in some detail how this bug can determine which of the three spaces he is living in.

3. Consider the following axiomatic system:
 - A1** Every ant has at least two paths.
 - A2** Every path has at least two ants.
 - A3** There exists at least one ant.
 - (a) (10 points) Prove that there must be at least two paths.

 - (b) (10 points) Define what is meant by a **complete** axiomatic system. Is the given system complete? Prove your answer.

 - (c) (10 points) Define what is meant by a **consistent** axiomatic system. Is the given system consistent? Give evidence to support your answer.

 - (d) (10 points) Define what is meant by an **independent** axiom in an axiomatic system. In the given system, is A1 independent? Prove your answer.

4. (20 points) Playfair's Postulate is:

Given a line and a point not on the line, it is possible to construct one and only one line through the given point parallel to the line.

Prove that the statement "whenever a line is perpendicular to one of two parallel lines, it must be perpendicular to the other" implies Playfair's Postulate. You may use Euclid's Postulates I-IV (and any theorems proved from these), but not Euclid's Postulate V.