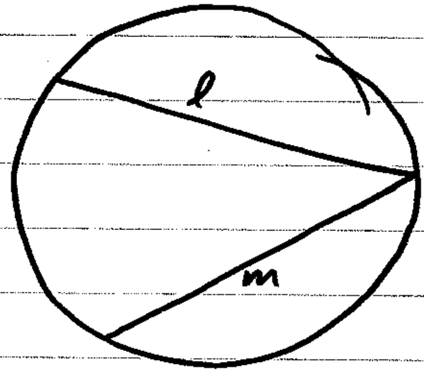
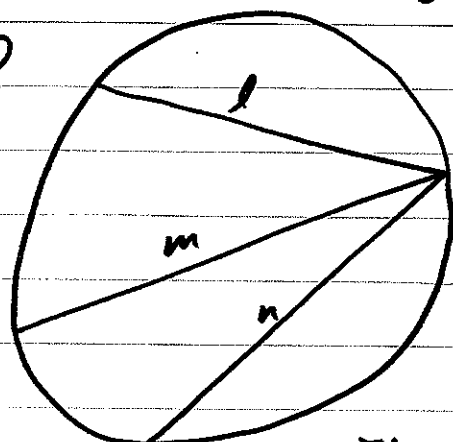
Math 402 hw solutions

7.3.1



In the Klein model, two lines are right-limiting parallels to each other if and only if they approach the same boundary point. So if l is right-limiting to m, then m is right limiting to l.

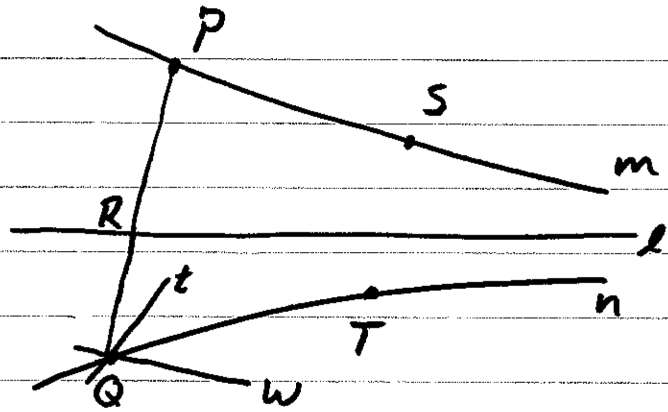
7.3.2



If l is right limiting to m and n is right limiting to n, then all three lines approach the same boundary point. Therefore l is also right-limiting to n.

7.3.3 Let m be right limiting to l. Then $r_l(l) = l$ and $r_l(m)$ is another line, which we denote by n. Since reflection preserves parallelism, n is parallel to l. To show that n is a right-limiting parallel to l, Let Q be a point on n. Drop a ~~parallel fro~~ perpendicular from Q to a point R on l. Let $P = r_l(Q)$. Then Q is on m and $\overline{QR} \perp l$ (reflection preserves angles). →

7.3.3 cont'd



Let S be on m and T on n as shown.

Let t be a line within angle $\angle RQT$. Then $r_l(t)$ is a line within angle $\angle RPS$. Since m is right-limiting to l, $r_l(t)$ intersects l. Therefore t intersects l (since a reflection sends intersecting lines to intersecting lines)

Let w be a line outside $\angle RQT$. Then $r_l(w)$ is outside $\angle RPS$. Since m is right-limiting to l, $r_l(w)$ does not intersect l. Therefore w does not intersect l (since a reflection sends parallel lines to parallel lines).

So n separates lines which intersect l from those which do not, so n is a right-limiting parallel to l.

~~An a~~ The right omega point of l is the set of all right-limiting parallels to l. The first part shows that r_l maps each of these limiting parallels to

7.3.3 cont'd

another limiting parallel, so it maps the set of right-limiting parallels of ℓ to itself. This means it fixes the right omega point of ℓ.

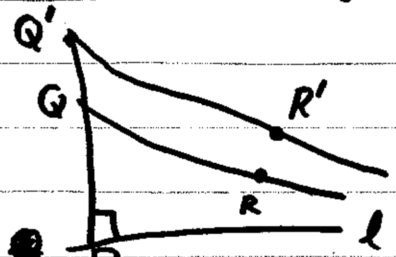
7.3.11 Let $\overline{PQ}$, ℓ, R be as described.

Let $\overline{P'Q'}$ also have length h, let ℓ' be $\perp$ to $\overline{PQ'}$ at Q', Let $\overleftrightarrow{P'R'}$ be the limiting parallel to ℓ' at P'.

This defines two omega triangles $\Delta PQ\Omega$ and ~~$\Delta P'\Omega' Q'$~~ $\Delta P'Q'\Omega'$.

Since $\overline{PQ} \cong \overline{P'Q'}$ and $\angle PQ\Omega \cong \angle P'Q'\Omega' = 90°$, Theorem 7.8 $\Rightarrow$ $\angle QP\Omega = \angle Q'P'\Omega'$. This is the angle $a(h)$. It is well defined because it depends only on h, not on the particular segment $\overline{PQ}$ which is chosen.

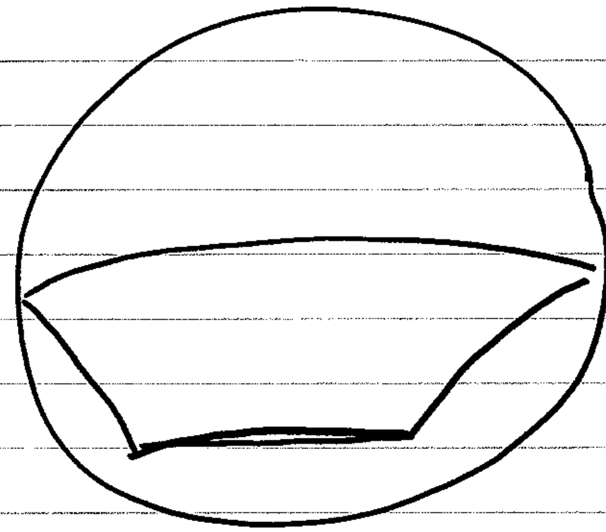
7.3.12 Let $\overline{PQ'}$ have length h' and find Q on $\overline{PQ'}$ so $\overline{PQ}$ has length h. Construct ℓ and limiting parallels $\overrightarrow{QR}$, $\overrightarrow{Q'R'}$ to ℓ



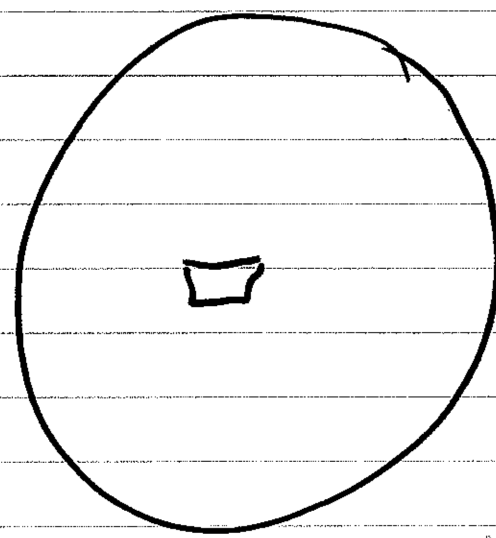
Then $\overline{QQ'}$, $\overrightarrow{QR}$, $\overrightarrow{Q'R'}$ form an omega Δ. By Exterior $\angle$ Theorem 7.7, $\alpha(h) > \alpha(h')$.

Geom Explorer Problems.

1. If you make a large Saccheri quad., you get the summit angles as close as you like to 0°.
My picture is poor!



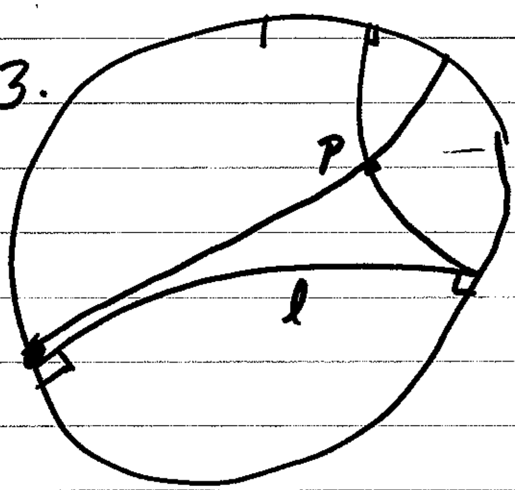
2. If you make a very small Saccheri quad., you get the summit angles as close as you like to 90°



Note: Summit angles never $= 0^\circ$ or 90°

$0^\circ <$ summit angle $< 90^\circ$

3.



not such a great picture!

4. From mathworld.wolfram.com

$$a(h) = 2\tan^{-1}(e^{-h})$$

Notice $\lim_{h \to 0} a(h) = 90^\circ$

$$\lim_{h \to \infty} a(h) = 0^\circ$$

On this one, just a chart is ok. I wouldn't expect them to get the function a(h)