# Math 403 - Test #3 - Review Questions

These questions certainly do not cover everything that could appear on the test, but should at least give you an idea of the types of questions to expect. I do not have written up for these!

The test will cover Sections 3.1-3.7, 4.1-4.2

1. All homework questions should always be considered possible test questions.
2. Give the definition of each of the following. You should give the definition used in our textbook or handouts.
   (a) scalar product of two vectors
   (b) length of a vector
   (c) distance from X to Y
   (d) orthogonal vectors
   (e) circumcenter of a triangle
   (f) altitudes of a triangle
   (g) orthocenter of a triangle
   (h) Euler line of a triangle (and what are the three special points which are on the Euler line?)
   (i) projection of $X$ onto line $l$ or onto vector $Y$.
   (j) isometry
   (k) linear isometry (answer: an isometry $\alpha$ with the property $\alpha(O) = 0$. See Theorem 4.3 for important properties of a linear isometry.)
   (l) collineation
   (m) fixed point (of an isometry)
3. Give an example of each of the following or explain why it cannot exist.
   (a) A vector $X$ such that $X \cdot X = -1$
   (b) A vector which is orthogonal to all vectors.
   (c) A triangle whose circumcenter is outside of the triangle.
   (d) A triangle whose orthocenter is outside of the triangle.
   (e) Two nonzero vectors $X$ and $Y$ such that $X \cdot Y = |X||Y|$.
   (f) A linear isometry with no fixed point.
   (g) A linear isometry with exactly two fixed points.
   (h) An isometry with an infinite number of fixed points.
   (i) An isometry having all points fixed.
4. Which of the following properties are preserved by all isometries?
   (a) straight lines
   (b) angles

(c) distance

(d) centroids

(e) midpoints of segments

(f) triangles

(g) parallel lines

(h) rectangles

5. Give the statement of the Nine-Point Circle Theorem and illustrate with a sketch.

6. State and prove the Cauchy-Schwarz Inequality.

7. Consider the central dilation $\delta_{C,r}$ with $r = 3$ and center $C = (1, 1)$. Find a vector $A$ and a linear isometry $\beta$ such that $\delta_{C,r} = \tau_A \circ \beta$.

8. Prove that if $\alpha$ is an isometry with fixed points $X$ and $Y$, then every point on the line through $X$ and $Y$ is fixed by $\alpha$. (you should be able to prove most of the theorems and propositions and sections 4.1 and 4.2).

9. True or False?

   (a) Every central dilatation sends a circle to a circle.

   (b) Every central dilatation sends a circle to a circle of the same radius.

   (c) If $d(X, Y) = 0$, then $X = Y$.

   (d) Let $a, b, c$ be the side lengths of a triangle and let $s = \frac{1}{2}(a+b+c)$. Then the area of the triangle is $\sqrt{(s-a)(s-b)(s-c)}$.

   (e) Every isometry has an inverse which is also an isometry.

   (f) Every isometry can be written as the composition of a translation and a linear isometry.

   (g) The set of all linear isometries is a group.

   (h) Every isometry is an affine transformation. (answer is True)

   (i) Every affine transformation is an isometry.

   (j) Every similarity is an isometry.

   (k) Every isometry is a similarity.