

Math 403 - Matrix Algebra Basics - February 23, 2007

This handout discusses 2x2 matrices only. All of it can be generalized to nxn matrices.

1. **Matrix Addition** $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} = \begin{bmatrix} a + a' & b + b' \\ c + c' & d + d' \end{bmatrix}$

2. **Scalar Multiplication** $k \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

3. **Matrix Multiplication** $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} = \begin{bmatrix} aa' + bc' & ab' + bd' \\ ca' + dc' & cb' + dd' \end{bmatrix}$

Notice that if we denote the rows of the first matrix by $\vec{r}_1 = [a, b]$, $\vec{r}_2 = [c, d]$ and the columns of the second matrix by $\vec{c}_1 = [a', c']$, $\vec{c}_2 = [b', d']$, then we can use the dot product to express matrix

multiplication: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} = \begin{bmatrix} \vec{r}_1 \cdot \vec{c}_1 & \vec{r}_1 \cdot \vec{c}_2 \\ \vec{r}_2 \cdot \vec{c}_1 & \vec{r}_2 \cdot \vec{c}_2 \end{bmatrix}$

4. **Identity Matrix** $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ The identity matrix behaves like "1" in multiplication; for any other 2x2 matrix A , $AI = A$ and $IA = A$.

5. **Determinant** The determinant of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the number $ad - bc$. Notation: $\det A = ad - bc$.

6. **Inverse Matrices** Given a 2x2 matrix A , if there is some matrix B with the properties $AB = I$ and $BA = I$, then A is called invertible and B is called the inverse of A . Notation: $B = A^{-1}$. Not all matrices are invertible! It can be proved that a matrix A has an inverse if and only if $\det A \neq 0$. A formula for the inverse of a 2x2 matrix is

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is } A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

7. Matrix Algebra has many familiar properties such as (capital letters denote matrices and small letters denote numbers):

- (a) $A + B = B + A$
- (b) $(A + B) + C = A + (B + C)$
- (c) $k(A + B) = kA + kB$
- (d) $(AB)C = A(BC)$
- (e) $A(B + C) = AB + AC$
- (f) $(k + m)A = kA + mA$
- (g) $(km)A = k(mA)$

However, some familiar algebraic properties are not true for matrices. Most notably, matrix multiplication is not commutative: AB is not always equal to BA . For some choices of A and B , AB equals BA , but in general not. For this reason, the order of multiplication is very important in the following formula for the inverse of a product:

If A and B are invertible, then AB is also invertible and

$$(AB)^{-1} = B^{-1}A^{-1}.$$