Name:____

Math 403 - Test #2 - March 12, 2007

Time: 50 minutes. Write your answers on the blank paper provided. Start a new page for each problem and be sure to number the problems. You may not use any books or notes. There are 100 points possible.

1. (a) (5 points) Give the definition of *affine transformation*.

   (b) (12 points) Determine whether or not each of the following is an affine transformation. You do not need to give a proof.

      i. $f(x, y) = (x, 0)$
      ii. $f(x, y) = (-x, -y)$.

   (c) Give an example of an affine transformation which has no fixed point. (Recall that P is called a fixed point of f if $f(P) = P$.)

2. (a) (5 points) Give the definition of a *group of transformations*.

   (b) (10 points) Consider all transformations of the form $f(X) = rX$, with $r \neq 0$. Do these form a group? Prove your answer.

   (c) (10 points) Give an example of a set of affine transformations which do not form a group. (Do not use part (b) above as your example!) Which group property does not hold for your example? A proof is not needed.

3. (a) (10 points) Give the statement of Ceva's Theorem.

   (b) (10 points) Give the statement of Menelaus' Theorem.

4. (3 points each part) Answer True or False for each part. You do not need to give an explanation and there is no partial credit.

   (a) Given any two triangles, there is an affine transformation taking one to the other.

   (b) A translation and a central dilatation do not in general commute.

   (c) If $\triangle ABC$ and $\triangle DEF$ are perspective with respect to point X and if f is any affine transformation, then $\triangle f(A)f(B)f(C)$ and $\triangle f(D)f(E)f(F)$ are perspective with respect to point $f(X)$.

5. (10 points) Explain why the following picture is impossible. Be sure to say which theorem you are using and explain how it is used.

6. (7 points) Draw two triangles which are perspective with respect to a point.

7. (7 points) Let f be the central dilatation with center C (marked on the sketch) and with ratio $r = 1/2$. Sketch $f(\triangle XYZ)$.