Name:____

Math 403 - Test #3 - April 13, 2007

Time: 50 minutes. Write your answers on the blank paper provided. Start a new page for each problem and be sure to number the problems. You may not use any books or notes. There are 100 points possible.

1. (7 points each part) Give the definition of each of the following. You don't need to give examples or additional properties, just the definition.
   (a) The orthocenter of a triangle.
   (b) An isometry.
   (c) A linear isometry.

2. (7 points each part) For each part, give an example or explain why no such example exists.
   (a) Two nonzero vectors X and Y with the property $|X + Y|^2 = |X - Y|^2$
   (b) Two vectors X and Y with $X \cdot Y = 12$, $|X| = 2$, $|Y| = 3$.
   (c) An isometry with exactly one fixed point.

3. (11 points) Give the statement of the Nine-Point Circle Theorem.

4. (a) (15 points) Prove that any given isometry can be written as the composition of a translation and a linear isometry.
   (b) (5 points) Let α be the central reflection with center $C = (1, 0)$. (So $\alpha(X) = -X + 2C$.) Find a vector A and a linear isometry β such that $\alpha = \tau_A \circ \beta$.

5. (15 points) Prove that an isometry maps lines to lines.

6. (3 points each part) Answer True or False for each part. You do not need to give an explanation and there is no partial credit.
   (a) If an isometry has three fixed points which are noncollinear, then it has infinitely many fixed points.
   (b) If a map f preserves dot products (i.e. $f(X) \cdot f(Y) = X \cdot Y$ for all X, Y), then f preserves lengths of vectors (i.e. $|f(V)| = |V|$ for all V).
   (c) The angles in plane geometry are completely determined by the scalar product (up to sign).
   (d) Given any two points X and Y, there is exactly one isometry α with the property that $\alpha(X) = Y$.