

Name: _____

Math 403 - Final Exam - May 7, 2007

Time: 3 hours. Write your answers on the exam. You may not use any books or notes. There are 110 points possible.

1. (10 points) Who was Euclid? About when and where did he live? What are Euclid's *Elements*? (Just a few sentences are enough)

2. (7 points) Let $A = (0, 0)$, $B = (1, 0)$, $C = (0, 1)$. Find the (x, y) coordinates of the centroid G of $\triangle ABC$. Show your work.

3. (5 points) Give the statement of Desargues Theorem.

4. (5 points) Give the definition of affine transformation.

5. (10 points) Prove that affine transformations preserve midpoints. That is, if f is an affine transformation and if Z is the midpoint of X and Y , then $f(Z)$ is the midpoint of $f(X)$ and $f(Y)$.

6. (15 points) State and prove the Cauchy-Schwarz inequality for vectors. (Note: if you've forgotten the statement, I can give that to you, but it will be -5 points)

7. (10 points) List all the types of isometries.

8. (10 points) Let $\tau(x, y) = (x, y + 2)$. Find two lines l and m so that $\tau = \sigma_l \circ \sigma_m$. (σ_l denotes reflection over l)

9. (7 points) Suppose f is an isometry which is not the identity and which fixes $(1, 1)$ and $(2, 2)$. Find $f(1, 0)$ and explain how you know $f(1, 0)$ must have this value.

10. (7 points) Suppose g is an isometry with $g(0, -1) = (1, 1)$, $g(1, 0) = (2, 0)$ and $g(2, 1) = (3, -1)$. Find $g(3, 3)$ and explain how you know $g(3, 3)$ must have this value.

11. (3 points each part) Answer True or False for each part. You do not need to give an explanation and there is no partial credit.

(a) Ceva's Theorem says the lines $\ell_{AA'}$, $\ell_{BB'}$ and $\ell_{CC'}$ are *parallel* if and only if

$$\frac{A' - B}{A' - C} \cdot \frac{B' - C}{B' - A} \cdot \frac{C' - A}{C' - B} = -1.$$

(b) All affine transformations preserve angle measure.

(c) In every axiomatic system, there must be undefined terms.

(d) Given noncollinear points A, B, C , the points with barycentric coordinates of the form $(1/2, b, c)$ form a line parallel to ℓ_{BC} .

(e) The set of all affine transformations form a *group*.

(f) Every affine transformation is also an isometry.

(g) The perpendicular bisectors of the sides of a triangle are always concurrent.

(h) All central dilatations are isometries.