MATH213 HW 6

Due Wednesday, October 4
Solve five of the six problems below.

1. There are 151 houses on a street. Each house has an address between 1 and 300, inclusive. Show that at least two houses have addresses that are consecutive integers.

2. There are 151 houses on a street. Each house has an address between 1 and 300, inclusive. Show that some two houses have addresses such that one of them is divisible by the other.

3. One hundred tickets, numbered $1, 2, \ldots, 100$ are sold to 100 different people for a drawing. There is a grand prize and three other smaller equal prizes. How many ways are there to award the prizes if
(a) there are no restrictions?
(b) the person holding ticket 1 wins one of the prizes?
(c) the people holding tickets 1 and 2 both win some prizes?
(d) the grand prize winner is a person holding ticket $1, 2$ or 3?

4. Find
(a) the coefficient of x^3y^7 in $(2x+y)^{10}$;
(b) the coefficient of $x^{13}y^{77}$ in $(3x-2y)^{90}$;
(c) a simpler formula for $\sum_{k=0}^{10}\binom{20}{k}\binom{15}{10-k}$.

5. Prove that
(a) $\sum_{k=0}^{20}\binom{20}{k}(-1)^k\,3^{n-k}2^k = 1$ for each positive integer n;
(b) $n\,3^{n-1} = \sum_{k=1}^{n}\binom{n}{k}k2^{n-k}$ for each positive integer n.

6. Find one binomial coefficient equal to the following expression

$$\binom{n}{k} + 3\binom{n}{k-1} + 3\binom{n}{k-2} + \binom{n}{k-3}.$$

(Hint: Use identities from Section 4.4.)