

MATH213 HW 8

Due Wednesday, October 18

Solve five of the six problems below.

1. What is the probability of these events when we randomly choose a permutation of $\{1, 2, \dots, n\}$ where $n \geq 4$?
 - (a) 2 precedes n .
 - (b) 1 precedes 2 and 2 precedes n .
 - (c) 1 immediately precedes 3.
 - (d) 1 immediately precedes 3, and n immediately precedes 2.
2. Suppose that E_1 and E_2 are events such that $p(E_1) = 3/4$ and $p(E_2) = 4/5$. Show that $11/20 \leq p(E_1 \cap E_2) \leq 3/4$.
3. A student takes a random card from a deck of 52 cards, looks at it and puts it back. If he does this k times, what is the probability that he took a card of some kind at least twice? What is the smallest k such that this probability is greater than $1/2$?
4. Suppose that a die is loaded so that 1 comes up twice as often as each other number. We roll the die three times. What is the expected sum of the numbers that come up?
5. Suppose we flip a coin until either it comes up heads two times in a row or we flipped it five times. What is the expected number of times we flip the coin?
6. An *inversion* in a permutation of $\{1, 2, \dots, n\}$ is an ordered pair (i, j) such that $i < j$ but j precedes i in this permutation. For example, the inversions in the permutation $(2, 4, 1, 3)$ are $(1, 2)$, $(1, 4)$, and $(3, 4)$. What is the expected number of inversions in a permutation of $\{1, 2, \dots, n\}$? Prove your answer.