MATH213 HW 9

Due Wednesday, October 25

Solve five of the first six problems below. For an additional credit, you can solve Problem 7.

1. The Fibonacci numbers, f_n, are defined as follows: $f_1 = f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for each $n \geq 3$. Prove that f_n is even if and only if n is divisible by 3.

2. Let h_n equal the number of different ways in which the squares of a $1 \times n$ chessboard can be colored using the colors white, red, and blue in such a way that no two squares that are colored red are adjacent. Find and verify a recurrence relation for h_n. Then find a formula for h_n.

3. Solve the recurrence relation $h_n = 4h_{n-2}$, $(n \geq 2)$ with initial values $h_0 = 0$ and $h_1 = 1$.

4. Solve the recurrence relation $h_n = 8h_{n-1} - 16h_{n-2}$, $(n \geq 2)$ with initial values $h_0 = -1$ and $h_1 = 0$.

5. Solve the nonhomogeneous recurrence relation $h_n = 4h_{n-1} + 3 \cdot 2^n$, $(n \geq 1)$ with the initial value $h_0 = 1$.

6. Solve the nonhomogeneous recurrence relation $h_n = 2h_{n-1} + n$, $(n \geq 1)$ with the initial value $h_0 = 1$.

7. What is the probability that a 13-card bridge hand has no Jacks, but has at least one King and at least one Queen? (It might have also cards of other values, but not Jacks.)