Course Description — SPRING 2010

# MATH 582
# STRUCTURE OF GRAPHS

**2pm MWF, Room 343 Altgeld Hall**

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TOPICS: This is a companion course to Math 581 – Extremal Graph Theory. The two courses are independent. Structure of Graphs includes topics drawn from the following (not all will be covered).

*Elementary Structural Concepts* – structural and enumerative topics involving trees and related graphs, degree sequences, embeddings of graphs in product graphs, and the reconstruction problem (is $G$ reconstructible from the deck of subgraphs obtained by deleting a single vertex? ... a single edge?). Graph packings and equitable colorings.

*Connection and Cycles* – min-max relations for connectivity and branchings, structure of $k$-connected graphs, Hamiltonian cycles and circumference, communication problems (gossip problem, etc.).

*Topological Graph Theory* – embeddings on surfaces (without edge crossings), characterizations and properties of graphs embeddable in the plane (separator theorems, Schnyder labelings), measures of non-planarity, voltage graphs and chromatic number of surfaces.

*Graph Minors* – treewidth and the minor order, some discussion of Robertson-Seymour Theorem (every minor-closed family of graphs has finitely many minimal forbidden minors), forbidden and forced minors.

*Matchings and joins* – the structure of graphs with perfect matchings, the structure of solutions of the Chinese Postman Problem, the language of conservative weightings for finding maximal joins and minimum $T$-joins, cycle covers and nowhere-zero flows.

*Algebraic Graph Theory* – eigenvalues of graphs (with application to expanders), graph polynomials, automorphisms, highly symmetric graphs.

COURSE REQUIREMENTS: There will be 8 problem sets, each requiring 5 out of 6 problems for 50 points total; no exam. The problems require proofs related to or applying results from class.

PREREQUISITES: Familiarity with elementary graph theory. Math 580 provides sufficient preparation, as do most versions of Math 412. Interested students with no graph theory background may browse a basic text in advance, such as Diestel, *Graph Theory*, or the Math 412 text: West, *Introduction to Graph Theory* (Prentice Hall, 2001, first seven chapters). Important results needed from elementary graph theory will be reviewed.

TEXT: D. B. West, *The Art of Combinatorics, Volume II: Structure of Graphs.* For some topics, instructor's supplements will be provided.