

MATH 582, SPRING 2010 – PROBLEM SET 7

Do five of the six problems below. Due Monday, May 3.

1. Complete the proof of Lemma 8.1.63.
2. Complete the proofs of Lemma 8.1.64, and Theorem 8.1.65.
3. Let T_1, T_2 , and T_3 be the three directed trees formed by the internal edges in a Schnyder labeling of a triangulation G . Let D be the digraph obtained by deleting the external edges and reversing the edges of T_1 . Prove that D has no directed cycles.
4. Let the “dimension” $\dim(G)$ of a graph G be the minimum number of linear orderings of the vertices such that for each vertex v and edge xy not incident to v , there is an ordering in the set in which v appears after both x and y . Prove that $\dim(K_{3,3}) > 3$ and $\dim(K_5) > 3$. (Hint: Show that some vertex never beats some non-incident edge.)
 5. a) Prove that every tree has a $(1, 2/3)$ -separation.
 - b) Prove that every outerplanar graph has a $(2, 2/3)$ -separation.
 - c) Prove that each grid with n vertices has a \sqrt{n} -separator with $\alpha = 1/2$.
6. A *normal plane map* is a connected plane multigraph in which all vertex and face degrees are at least 3. Prove that every normal plane map has an edge with the sum of the degrees of the ends at most 13. Give an example that this is sharp. (Hint: Reduce the problem to triangulations and use discharging but beware that you can add an edge xy only if $d(x) + d(y) \geq 12$.)