1. Find a good eulerization of the following graph and show that it is the best one.

x, y, w, z are vertices of odd degree. So to eulerize the graph we pair them up:

- xy, wz needs 2 + 3 extra edges
- xw, yz needs 4 + 3 extra edges
- xz, yw needs 5 + 2 extra edges

Hence we need 5 extra edges to eulerize it efficiently.
(2) Consider the graph below

(i) find a small cost Hamiltonian cycle by nearest-neighbors algo., starting at A
(ii) find a small cost Hamiltonian cycle by sorted-edges algo.
(iii) find a minimum cost spanning tree in the graph.

(i) Starting from A we choose the edges in the following order:

\[ AB - BD - DC - CE - EA \]

Total cost: \[2 + 3 + 1 + 3 + 9 = 18\]

(ii) Here we choose the edges in the following order:

\[ DC, AB, BD, EC, EA \] : Total cost = 18

(iii) The edges in order are:

\[ DC, AB, BD, EC \] : Total cost = 9
Consider the following election with 22 voters and 4 candidates.

First winners by:
(i) Condorcet's rule
(ii) Borda count
(iii) Hare's System.

<table>
<thead>
<tr>
<th>Rank</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>C</td>
<td>B</td>
<td>D</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>D</td>
<td>A</td>
<td>B</td>
<td>B</td>
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<tr>
<td>C</td>
<td>D</td>
<td>A</td>
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<td>D</td>
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</tr>
<tr>
<td>D</td>
<td>A</td>
<td>C</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

(i) A vs B: 8 : 14 : A < B, A cannot win
B vs C: 12 : 10 : C < B, C cannot win
B vs D: 10 : 12 : B < D, B cannot win
D vs A: 10 : 12 : D < A, D cannot win

Hence no Condorcet winners.

(ii) Score of A: 0 + 0 + 24 + 8 = 32
   B: 4 + 12 + 8 + 12 = 36
   C: 8 + 18 + 0 + 4 = 30
   D: 12 + 6 + 16 + 0 = 34

B wins.

(iii) In the first round B and D are ruled out
and then the tally is as follows:

<table>
<thead>
<tr>
<th>4</th>
<th>6</th>
<th>8</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>C</td>
<td>A</td>
<td>A</td>
</tr>
</tbody>
</table>

Hence A wins C by 12 : 10.
X and Y are moving on after being roommates for four years in college. They must divide the following items among themselves.

<table>
<thead>
<tr>
<th>Object</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bike</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Textbooks</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>Barbells</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Rowing Machine</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Music Collection</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>Computer</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>Novels</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>Desk</td>
<td>15</td>
<td>10</td>
</tr>
</tbody>
</table>

\[
X: 20 + 5 + 15 = 40 \\
Y: 12 + 10 + 11 + 17 + 22 = 72
\]

\[
\begin{align*}
A & : 12/10 = 1.2 \\
B & : 10/7 = 1.42 \\
E & : 11/8 = 1.375 \\
F & : 17/15 = 1.13 \\
G & : 22/20 = 1.1 \\
\end{align*}
\]

Hence we move G 6, but must move only X fraction of G.

Hence X: \(40 + 20x\)
\[
Y = 72 - 22x
\]

Equating them we get \(x = \frac{16}{21}\)

Hence X has B, C, H, \(\frac{16}{21} G\)

Y has A, D, E, F, \(\frac{16}{21} G\).

(i) Any graph with all degrees even has an Euler circuit.

(ii) A graph can have an Euler circuit but no Hamiltonian circuit.

(iii) Sequential Pairwise Voting does not satisfy CWC.

(iv) Minimum cost of a spanning tree can be greater than the minimum cost of a Hamiltonian cycle in a graph.

(i) False, only if the graph is connected.

(ii) True, for example \[ \xrightarrow{\text{has an Euler circuit}} \] has an Euler circuit but no Hamiltonian circuit.

(iii) False. It satisfies CWC. Assume that A wins by Condorcet's rule. It means that A beats every other candidate. So if the election is done by Seq. Pairwise, then A will beat the rest when his turns comes and will be the only one who survives.

(iv) False. In a min cost ham. cycle, if we delete any edge, then what remains is a tree and clearly has lower cost than the ham. cycle.