Name: Solutions

Quiz 1

1. Let \( \vec{a}_1 = \left( \frac{1}{2} \right), \vec{a}_2 = \left( \frac{3}{6} \right), \vec{a}_3 = \left( \frac{-4}{3} \right), \ vec{b} = \left( \frac{4}{1} \right) \).

Determine if \( \vec{b} \) is in \( \text{Span} \{ \vec{a}_1, \vec{a}_2, \vec{a}_3 \} \).

Sol: Form \( \begin{bmatrix} 4 & b \end{bmatrix} \)

\[
\begin{bmatrix}
  1 & 0 & -4 & 4 \\
  0 & 3 & -2 & 1 \\
  2 & 6 & 3 & -4
\end{bmatrix}

\xrightarrow{R_3 = R_3 - 2R_1}

\begin{bmatrix}
  1 & 0 & -4 & 4 \\
  0 & 3 & -2 & 1 \\
  0 & 6 & 11 & -12
\end{bmatrix}

\xrightarrow{R_3 = R_3 - 2R_2}

\begin{bmatrix}
  1 & 0 & -4 & 4 \\
  0 & 3 & -2 & 1 \\
  0 & 0 & 15 & -14
\end{bmatrix}

This is in row echelon form and clearly has a solution. Hence \( \vec{b} \) is in \( \text{Span} \{ \vec{a}_1, \vec{a}_2, \vec{a}_3 \} \).
(ii) State True/False, and justify.

(i) If $S$ is a set of vectors in $\mathbb{R}^m$, then $|S| \leq m$

(iii) Is the converse of the above statement true?

Not true.

(i) True; if we form a matrix $A$ with columns as the vectors in $S$, then it has more columns than rows. Hence while solving $A\tilde{x} = \tilde{b}$, we will have at least one free variable, and thus linearly dependent, a contradiction.

(iv) No. Consider the set $S = \{(1, 0, 1), (0, 1, 2)\}$

Here $m = 3$ and $|S| = 3$, but it's not a linearly independent set since

$$1 \cdot \left(\frac{2}{3}\right) + 1 \cdot \left(\frac{1}{3}\right) - 2 \cdot \left(\frac{2}{3}\right) = \left(\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right)$$