

Math 242
Test 1 1.09, VC.01 – VC.03
Fall 2005

■ Name: _____

Show all work on this exam. Work must support answers.

1) Given vectors $X = \{2, 3, -1\}$ and $Y = \{1, 2, -3\}$ (5 points each)

a) Find a vector of length 5 that is parallel to X.

b) Find an equation of a line that is parallel to X and passes through the point $P = \{-3, 4, 0\}$

c) Find an xyz– equation of the plane that passes through P and contains the vectors X and Y.

d) Find $X \cdot Y$.

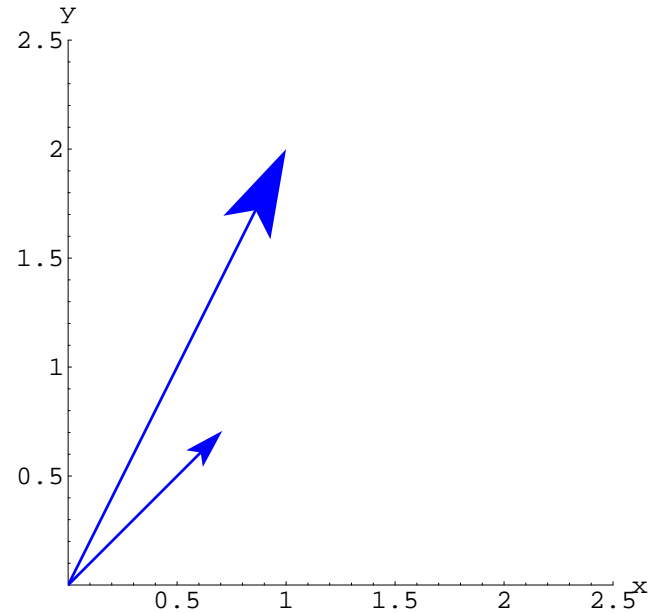
e) Is the angle between the vectors X and Y acute, obtuse or right?

2) Give the number t that makes $X = \{3, -1, 2\}$ and $Y = \{1, 1, t\}$ perpendicular. (6 points)

3) The component of a vector X in the direction of another vector Y is given by $(\frac{X \cdot Y}{Y \cdot Y}) Y$.

What does this formula reduce to if Y is a unit vector? (6 points)

4) Here are the vectors $\mathbf{X} = \left\{ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\}$ and $\mathbf{Y} = \{1, 2\}$ with their tails at $\{0, 0\}$, shown in true scale:



Identify X and Y, and pencil in the component of Y in the direction of X.
(8 points)

Calculate the component of Y in the direction of X.

5) Given two 3D vectors X and Y, say why you are sure that
(6 points)

$$\mathbf{X} \cdot (\mathbf{X} \times \mathbf{Y}) = \mathbf{Y} \cdot (\mathbf{X} \times \mathbf{Y}) = 0.$$

6) A plane has xyz-equation (6 points)

$$x - 2y + 7z = 14.$$

Give a vector perpendicular to the plane, and give a point on the plane.

7) Given a fixed point $\{a, b, c\}$, and fixed, perpendicular unit vectors X and Y in 3D, describe the curve traced out by

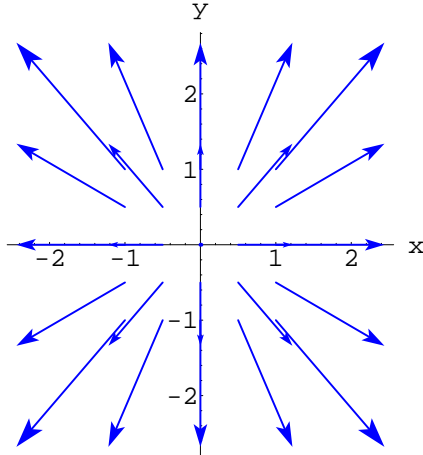
$$P[t] = \{a, b, c\} + 2 \cos[t] X + 2 \sin[t] Y \text{ as } t \text{ runs from } 0 \text{ to } 2\pi.$$

(6 points)

8) Go with

(6 points)

$f[x, y] = 0.6 (1 + 1.2 x^2 + 1.4 y^2)$,
and look at a plot of $\text{grad}f[x, y]$ with tails at $\{x, y\}$ for some selected points $\{x, y\}$ in the vicinity of $\{0, 0\}$:



Why are these gradient vectors so repelled by the point $\{0, 0\}$?

9) Go with

(6 points)

$$f[x, y, z] = x y \text{Cos}[z].$$

In which direction should you leave the point $\{2, 1, 0\}$ to get the greatest possible initial increase of $f[x, y, z]$?

10) Does (6 points)

$$f[x, y] = \frac{\sin[2x] + \sin[3y]}{0.5 + x^2 + y^2}$$

have a maximizer or minimizer? _____

How do you know?

11) When you are trying to find the highest crest or the deepest dip on the plot of a surface $z = f[x, y]$, why do you become interested in solving $\text{grad}f[x, y] = \{0, 0\}$ for x and y ? (6 points)

12) Explain why if X and Y are two perpendicular unit vectors, is $X \times Y$ also a unit vector. (6 points)

If X and Y are not perpendicular, is this still true?

13) How is the total differential related to this:
(6 points)

$$\nabla f[x, y] \cdot \{dx, dy\} = \left\{ \frac{\partial f[x, y]}{\partial x}, \frac{\partial f[x, y]}{\partial y} \right\} \cdot \{dx, dy\}?$$

14) Here is

(6 points)

$$\frac{\partial f[x, y]}{\partial x} = f^{(1,0)}[x, y]$$

for

$$f[x, y] = e^{\sin[x y - x]};$$

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Clear[f, x, y];
f[x_, y_] = E^Sin[x y - x];
D[f[x, y], x]
-E^-Sin[x - x y] (1 - y) Cos[x - x y]
```

And here is

$$\frac{\partial f[2.3, 8.4]}{\partial x} = f^{(1,0)}[2.3, 8.4]$$

for the same function:

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D[f[x, y], x] /. {x -> 2.3, y -> 8.4}
-0.72017
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What does this calculation tell you about what happens to $f[x, y]$ when you hold y at 8.4, but make x just a teensy-weensy bit bigger than 2.3?