

## Math 242 Test 2 Study Guide

### VC.04 - VC.09

This guide lists the important concepts from each of the lessons that you should be comfortable with prior to taking the test. If you have any uncertainties about these concepts, you should refer back to the appropriate Basics, Tutorials or GiveItaTry sections. The best way to test your understanding is to work out the problems from the Literacy Sheets of each lesson. Also, retaking the Mallard quizzes may be beneficial.

#### VC.04 2D Vector Fields and Their Trajectories

- Visualizing vector fields and identifying sources and sinks.
- Trajectories in a vector field and relationship to field vectors, sources and sinks.
- Flow along and across a curve and determining net flow.
- Direction of tangent and normal vector with respect to direction of parametrization.
- Component of a vector field along the tangent and along normal vector.
- Gradient fields and behavior of its trajectories.
- Differential equations and their associated vector fields.

#### VC.05 Flow Measurements by Integrals

- Measuring net flow of  $\text{Field}[x,y]$  along and across a curve  $\{x[t],y[t]\}$  for  $a \leq t \leq b$ .

$$\text{⚡ Flow along: } \int_a^b \text{Field}[x[t], y[t]] \cdot \{x'[t], y'[t]\} dt$$

$$\text{⚡ Flow across: } \int_a^b \text{Field}[x[t], y[t]] \cdot \{y'[t], -x'[t]\} dt$$

- Calculation and interpretation of path integrals over a curve.
- Directed curves, path independence and gradient fields.
- Identifying sources and sinks with flow across integral.
- Singularity sources and sinks.
- Measuring work done by a force field along a curve.

#### VC.06 Sources, Sinks, Swirls and Singularities

- Sources, sinks, and the divergence of a vector field
- Rotation (swirl) of a vector field
- Using a 2D integral to measure flow across/along closed curves.
- Flow calculations in the presence of singularities.
- The significance of  $\text{divField}[x,y] = 0$ .
- The significance of  $\text{rotField}[x,y] = 0$ .
- Singularity sources, sinks and swirls.
- The Laplacian of a function  $f[x,y]$ ,  $\frac{\partial^2 f[x,y]}{\partial x^2} + \frac{\partial^2 f[x,y]}{\partial y^2}$ .

**VC.07 Transforming 2D Integrals**

- Transforming 2D integrals: How you do it and why you do it.
- The area conversion factor  $A_{xy} [u, v]$
- Transforming  $\iint_{R_{xy}} f [x, y] \, dx \, dy$  when the boundary of  $R_{xy}$  is given by parametric formulas.
- Transforming  $\iint_{R_{xy}} f [x, y] \, dx \, dy$  when the boundary of  $R_{xy}$  is not given with parametric formulas.
- Flow measurements through integral transformations.
- What can happen when  $A_{xy} [u, v] = 0$ ? What sign of  $A_{xy} [u, v]$  tell you?
- Switching the order of integration.

**VC.08 Transforming 3D Integrals**

- 3D integrals.
- Integral transformations in 3D.
- The volume conversion factor  $V_{xyz} [u, v, w]$ .
- Plotting and integrating on cylinders, spheres, and tubes.
- Integrating on solids bounded by sets of surfaces
- Switching the order of integration in 3D.
- Measuring volume of a solid using 3D integral

**VC.09 Spherical Coordinates**

- Spherical coordinates (know the coordinates).
- Integrating with spherical coordinates (know the volume conversion factor).
- Plotting with spherical coordinates.
- Cylindrical coordinates (polar coordinates in 3D).