Math 242 Test 1 05 - 10 - 2005

Instructor: Bill Hart Name: .....

Time Allowed: 50 minutes. TA's Name: .....

Instructions: Complete **all** answers in pen in the space provided. You may write on both sides of the sheet. Show **all** working. If you require additional paper during the exam, please raise your hand.

Section A : FIVE Short Answer Questions - FIVE POINTS TOTAL

Section B : THREE Long Answer Questions - TEN POINTS TOTAL

Section A

Short Answer Questions

Question 1: (1 Point)

Convert the following equation (given in spherical coordinates) to:

i) Cylindrical coordinates; and

ii) Cartesian coordinates

$$\rho = \sin\phi\cos\theta + \sin\phi\sin\theta - 2\cos\phi.$$

Question 2: (1 Point)

i) Find the equation for the level curve corresponding to $z = 3$ for the surface with equation

$$\frac{z^2}{4} = \frac{x^2}{3} + \frac{y^2}{9} + 1. \tag{1}$$

ii) What is the name of the surface with equation given by (1) and what is the name of the level curve you have found.

Short Answer Questions (continued)

Question 3: (1 Point)

Find a unit vector of the form (a, b, b) for some $a, b \in \mathbb{R}$, which is perpendicular to the vector $(2, 1, 1)$.

Question 4: (1 Point)

Find an equation of the plane passing through the points $P = (1, 1, 0)$, $Q = (0, 0, 1)$ and $R = (1, 0, 0)$.

Short Answer Questions (continued)

Question 5: (1 Point)

Find the velocity vector, speed and acceleration vector (at time t) of a particle with position vector given by

$$\mathbf{r}(t) = (t^2, 3te^t).$$

Section B

Long Answer Questions

Question 6: (3 Points)

Find the angle between the planes in $\mathbb{R}^3$ with equations

$$2x - y + z = 5 \quad \text{and} \quad x + y - z = 1.$$

Long Answer Questions (continued)

Question 7: (4 Points)

Compute the curvature of the following curve and find the point on the curve where the curvature is maximum:

$$y = x^2 - 4x + 6.$$

Long Answer Questions (continued)

Question 8: (3 Points)

a) Prove that the line of intersection of the planes with equations

$$x + 2y - z = 1 \quad \text{and} \quad 3x + 2y + 2z = 4$$

is parallel to the line with parametric equations

$$x = 2t + 3, \quad y = t + 1, \quad z = 4t + 2.$$

b) Find an equation of the plane determined by the two parallel lines mentioned in part a).