Math 242 Test 3 07 - Dec - 2005

Instructor: Bill Hart Name: ....

Time Allowed: 50 minutes. TA's Name: ....

<u>Instructions</u>: Write your answers in pen. You may write on both sides of the sheet. If you require additional paper during the exam, please raise your hand.

For **all** questions, show all your computations and give *sufficient indication* of how they relate to the solution. In particular, *long answer* questions will not receive credit unless your solutions include detailed explanations.

Submit all rough work. Place a single diagonal line through any rough work which you do not consider to be part of your final solution.

All students should aim to complete BOTH the long answer questions and as many of the short answer questions as they have time for. Complete, correct solutions may attract more marks than a larger number of incomplete or incorrect solutions.

Section A : FIVE Short Answer Questions - TWENTY SEVEN POINTS TOTAL

Section B : TWO Long Answer Questions - EIGHTEEN POINTS TOTAL

Section A

Short Answer Questions

Question 1: (6 Points)

Find the volume of the region under the surface $z = x$ and over the region R in the xy-plane bounded by $y = 0$ and $y = \sin x$, $0 \leq x \leq \pi$.

Question 2: (5 Points)

Sketch the region of integration of the integral

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} \, dy \, dx.$$

Now change the order of integration and evaluate the resulting integral.

Short Answer Questions (continued)

Question 3: (5 Points)

Use *double* integration to find the area of the region bounded by

$$y = x^2 \text{ and } y = \frac{2}{1+x^2}.$$

Question 4: (5 Points)

Find the volume of the "ice cream cone" shaped solid bounded by the sphere $x^2 + y^2 + z^2 = 1$ on top and the cone $z = \sqrt{x^2 + y^2}$ on the bottom. [Hint: use double integration in polar coordinates.]

Question 5: (6 Points)

Find the value of the triple integral

$$\int\int\int_T f(x, y, z)\, dV,$$

where $f(x, y, z) = x^2$ and T is the tetrahedron bounded by the coordinate planes and the first octant part of the plane with equation $x + y + z = 1$.

Section B

Long Answer Questions

Question 6: (9 Points)

Approximate the integral

$$\int\int_R (4x^3 + 6xy^2)\, dA,$$

over the rectangular region $R = [1, 3] \times [-2, 1]$, by means of a Riemann sum involving a partition of R into six squares, R_i, $1 \leq i \leq 6$, each having side length 1. Take for your selection, the points (x_i^*, y_i^*) in the *lower left* corner of each of the squares R_i (assuming the positive y-axis is oriented up and the positive x-axis is oriented to the right of the page). Now compute the exact value of the double integral using an iterated integral.

Long Answer Questions (continued)

Question 7: (9 Points)

Compute the mass and centroid of the plane lamina bounded by $y = 0$ and $y = \sin x$ for $0 \leq x \leq \pi$ given that it has density given by the function $\delta(x, y) = x$.