Sample Test 3

1. Find the value of the triple integral

$$\iiint_T f(x, y, z)\, dV,$$

where $f(x, y, z) = x^2$ and T is the tetrahedron bounded by the coordinate planes and the first octant part of the plane with equation $x + y + z = 1$.

2. Compute the mass and centroid of the plane lamina bounded by $y = 0$ and $y = \sin x$ for $0 \leq x \leq \pi$ given that it has density given by the function $\delta(x, y) = x$.

3. Find the volume of the solid under the surface

$$z = \frac{xy}{1 + x^2y^2}$$

and over the region bounded by $xy = 1, xy = 5, x = 1$, and $x = 5$.

4. Compute the integral

$$\iiint_R z(x^2 + y^2)^{(-1/2)}\, dx\, dy\, dz,$$

where R is the region bounded above by the plane $z = 2$ and below by the surface $2z = x^2 + y^2$.

5. Find the amount of work done moving in a straight line from $(1, 0, 2)$ to $(3, 4, 1)$ in the force field $\mathbf{F}(x, y, z) =\rangle 2xy, x^2 + 2, y\rangle$.

6. Use the fundamental theorem of line integrals to compute

$$\int_C \langle y, x\rangle \cdot \mathbf{T}\, ds,$$

where C is any path from $(0, 0)$ to $(2, 4)$.

7. Use Green's Theorem to find the work done by the force field $\mathbf{F}(x, y) = \langle 3y - 4x, 4x - y \rangle$ when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

Challenge! Let $\mathbf{F}(x, y) = \langle x^2 y, xy^3 \rangle$. Minimize the amount of work done to move from $A(0, 0)$ to $B(1, 1)$ and find the path that minimizes this work. Hint: The path from A to B can be expressed in the form $\mathbf{r}(t) = \langle x(t), y(t) \rangle$, where $x(t)$ and $y(t)$ are both power series. What restrictions are there on the variables?