Sample Test 1 Solutions

1. a) $\vec{x}$ has length $\sqrt{14}$, so $\frac{5}{\sqrt{14}}\vec{x}$ has length 5.
b) $L(t) = \{-3, 4, 0\} + t\{2, 3, -1\}$.
c) $\vec{x}\times\vec{y} = \{-7, 5, 1\}$, so the equation of the plane is $-7(x+3)+5(y-4)+z = 0$ or $-7x + 5y + z = 41$.
d) $\vec{x} \cdot \vec{y} = 2 + 6 + 3 = 11$
e) Since $\vec{x} \cdot \vec{y} > 0$, the angle is acute. (Recall the dot product formula.)

2. We want $\vec{x} \cdot \vec{y} = 0$, so $\vec{x} \cdot \vec{y} = 3 - 1 + 2t = 0$, and $t = -1$.

3. If $\vec{y}$ is a unit vector, then $|\vec{y}|^2 = \vec{y}\cdot\vec{y} = 1$, so the formula reduces to $(\vec{x}\cdot\vec{y})\vec{y}$.

4. Note that $\vec{x}$ is a unit vector, so applying the result of the previous problem, the component of $\vec{y}$ in the $\vec{x}$ direction is $(\vec{x} \cdot \vec{y})\vec{x} = \left(\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}}\right)\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\} = \left\{\frac{3}{2}, \frac{3}{2}\right\}$.

5. Since $\vec{x} \times \vec{y}$ is perpendicular ot both $\vec{x}$ and $\vec{y}$, we know that both dot products are zero.

6. The normal vector is $\{1, -2, 7\}$ and one point is $\{14, 0, 0\}$. There are an infinite number to choose from.

7. $P(t)$ traces out a circle of radius 2, centered at $\{a, b, c\}$, and in the plane parallel to $\vec{x}$ and $\vec{y}$ (or normal to $\vec{x} \times \vec{y}$).

8. Since gradient vectors point in the direction of greatest increase, $\{0, 0\}$ is a minimum.

9. $\nabla f = \{y\cos(z), x\cos(z), -xy\sin(z)\}$, so $\nabla f(2, 1, 0) = \{1, 2, 0\}$.

10. Yes, is has both a maximum and a minimum. We know the function is bounded between -4 and 4 and oscillates. As $x, y \to \pm\infty$, $f(x, y) \to 0$, so there will be a maximum and a minimum near the origin.

11. Highest crests and deepest dips have horizontal tangent planes, so at those points we have $f_x = f_y = 0$, i.e. $\nabla f = \{0, 0\}$.

12. $|\vec{x} \times \vec{y}| = |\vec{x}||\vec{y}||\sin(\theta)| = 1$. If $\vec{x}$ and $\vec{y}$ are not perpendicular, this is not true, since then $0 \leq \sin(\theta) < 1$.

13. The total differential is $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = \nabla f \cdot \{dx, dy\}$.

14. f decreases slightly.

15. This will not be covered on the exam, but if you're curious, the net flow across turns out to be 8π, so the flow is inside to outside.

16. This will not be covered on the exam, but if you're curious, the net flow along the circle turns out to be 0, so there is no net flow along the curve.

17. This means $f(x, y)$ is a local maximum. Gradient vectors will therefore point towards it and thus outside to inside the circle C.

18. The field vectors are tangent to the trajectory in the direction of motion. For this reason, two trajectories cannot cross, since otherwise you would have two distinct field vectors defined at the point of intersection, an impossibility. [Insert Mr. Yuckface.]