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Homological Algebra
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1. a) $\frac{5\{2,3,-1\}}{\sqrt{14}}$
b) $r(t) = \{-3, 4, 0\} + t\{2, 3, -1\}$
c) $-7(x + 3) - 5(y - 4) + z = 0$
d) $2 + 6 + 3 = 11$
e) acute

2. $3 - 1 + 2t = 0 \Rightarrow t = -1$.

3. $(X \cdot Y)Y$.

4. Component of Y in the direction of X is $\frac{X \cdot Y}{X \cdot X} X = \left(\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}}\right) X = \frac{3\sqrt{2}}{2} X = \{3/2, 3/2\}$.

5. Since $X \times Y$ is perpendicular to both X and Y and since the dot product of any two perpendicular vectors is 0, we know that $X \cdot (X \times Y) = Y \cdot (X \times Y) = 0$.

6. Perpendicular vector $= \{1, -2, 7\}$
Point on the plane $= \{0, 0, 2\}$ (There are an infinite number of answers to this.)

7. This is a circle of radius 2 centered at the point $\{a, b, c\}$.

8. The function $f(x, y)$ has a global minimum at the origin and increases without bound as $x, y \to \infty$. Cross sections of the paraboloid are ellipses that are almost circles, so the gradient will almost be pointing directly away from the origin.

9. $\nabla f(2, 1, 0) = \{y \cos z, x \cos z, -xy \sin z\}(2, 1, 0) = \{1, 2, 0\}$

10. Yes. The denominator goes to infinity as $x, y \to \infty$, so $f(x, y) \to 0$ as $x, y \to \infty$. The numerator, on the other hand, oscillates between positive

and negative values, whose magnitude will be as high as 2. So somewhere near the origin, this function is certain to have a global maximum and global minimum.

11. The solutions to $grad(f(x, y)) = \{0, 0\}$ give us the locations of local maxima and minima. Evaluating f at these locations and comparing them tells us where the global maximum (highest crest) and global minimum (deepest dip) are.

12. If X and Y are perpendicular unit vectors, then $|X \times Y| = |X||Y| \sin \pi/2 = 1$, so $X \times Y$ is a unit vector. If X and Y are not perpendicular, then $X \times Y$ is not a unit vector because $-1 < \sin \theta < 1$, so $0 \leq |X \times Y| < 1$.

13. The total differential df is precisely $\nabla f(x, y) \cdot \{dx, dy\}$, as seen by expanding out the dot product.

14. Since $\frac{\partial f}{\partial x}(2.3, 8.4) < 0$, we know that if we move in the direction of increasing x from $(2.3, 8.4)$, we will at least initially go down on the surface $z = f(x, y)$. So making x a teensy-weensy bit bigger, $f(x, y)$ gets a little smaller.