

Solutions to Sample Exam 1

1. Multiply through by ρ , and you immediately have the Cartesian equation:

$$x^2 + y^2 + z^2 = x + y - 2z.$$

From this you get the cylindrical equation:

$$r^2 + z^2 = r \cos \theta + r \sin \theta - 2z.$$

By completing the square in the Cartesian equation, you find that this is a sphere of radius $\sqrt{3/2}$ centered at $(1/2, 1/2, -1)$.

2. (i) $\frac{x^2}{3} + \frac{y^2}{9} = \frac{5}{4}$.

(ii) The level curve is an ellipse. The 3-D surface is an elliptic hyperboloid of two sheets with vertices at $(0, 0, \pm 2)$.

3. $\frac{\langle 1, -1, -1 \rangle}{\sqrt{3}}$.

4. $\vec{PQ} = \langle -1, -1, 1 \rangle$ and $\vec{PR} = \langle 0, -1, 0 \rangle$ form a basis for the plane, so $\vec{n} = \langle -1, 0, 1 \rangle$. The equation of the plane is then $-x + z = -1$.

5. $\vec{v}(t) = \langle 2t, 3e^t + 3te^t \rangle$
 $v(t) = \sqrt{4t^2 + 9e^{2t} + 12te^{2t} + 9t^2e^{2t}}$
 $\vec{a}(t) = \langle 2, 6e^t + 3te^t \rangle$

6. $\vec{n}_1 = \langle 2, -1, 1 \rangle$
 $\vec{n}_2 = \langle 1, 1, -1 \rangle$
 $\vec{n}_1 \cdot \vec{n}_2 = 0$. Since the normal vectors are orthogonal the angle between the two planes is $\pi/2$.

7. $\vec{r}(t) = \langle t, t^2 - 4t + 6, 0 \rangle$
 $\vec{v}(t) = \langle 1, 2t - 4, 0 \rangle$
 $\vec{a}(t) = \langle 0, 2, 0 \rangle$
 $v(t) = \sqrt{1 + 4t^2 - 16t + 16}$
 $|\vec{v}(t) \times \vec{a}(t)| = |\langle 0, 0, 2 \rangle| = 2$
 $\kappa = \frac{2}{\sqrt{(1+4t^2-16t+16)^3}}$.

The point of highest curvature on a parabola is at the vertex. $y' = 2x - 4 = 0$, so the point of highest curvature is $(2, 2)$.

8. The direction of the line is the cross product of the normal vectors of the planes, which is $\langle 6, -5, -4 \rangle$, which is not at all parallel to the given line. So part (b) is impossible.