Test 1 Solutions

1. a) $\frac{7}{3}\{1,-2,2\}$
b) $L(t)=\{-2,3,1\}+t\{1,-2,2\}$
c) $\vec{x}\times\vec{y}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 2 \\ 2 & 1 & -1 \end{vmatrix}=\{0,5,5\}$ So the equation of the plane is $z+z=4$.
d) 2
e) obtuse

2. $\begin{cases} x(t) &= 1 \\ y(t) &= 6+4t \\ z(t) &= 3-7t \end{cases}$

3. The dot product of the two direction vectors is 0 and they intersect at $\{6,0,1\}$

4. $\frac{\vec{x}\cdot\vec{x}}{\vec{y}\cdot\vec{y}}\vec{y}$ is the projection of $\vec{x}$ onto $\vec{y}$, thus the angle at the tip of $\frac{\vec{x}\cdot\vec{x}}{\vec{y}\cdot\vec{y}}\vec{y}$ is a right angle.

5. The intersection is a line. To find an equation for the intersection, find a point common to both planes, and cross the two normals to find the direction.

6. Their normals are perpendicular.

7. $P(t)$ traces a circle of radius 3 centered at $\{1,3,7\}$ in the plane parallel to (or containing) $\vec{x}$ and $\vec{y}$.

8. The flow is outside to inside since the point $\{x_0,y_0,f(x_0,y_0)\}$ is a local maximum, so the gradient vectors generally point towards it, and specifically in the direction of greatest increase of f.

9. $\nabla f=\{y\sin z, x\sin z, xy\cos z\}$, so $\nabla f(-1,2,\pi/2)=\{2,-1,0\}$.

10. Yes. As $x,y\to\pm\infty$, $f(x,y)\to 0$, and $0<e^{-(x^2+y^2)}\leq 1$, so depending on the behavior of x^6+5y^3+1, there will be a maximum and minimum near the origin.

11. The highest crest and deepest dip of a surface are the points at which

$\nabla f = \{0, 0\}$, that is, where the surface has a horizontal tangent plane.

12. $unittan(t)$ and $mainunitnormal(t)$ are unit vectors that are perpendicular to each other, so their cross product is perpendicular to both and is itself a unit vector.

13. No. Consider the two different tangent vectors at the point of intersection.

14. The net flow is clockwise and outward.