Math 285 Homework #12

Due Friday 7 May, 4:00pm, to my office 376 Altgeld

- Section 9.6 #21 *Comment.* The factor of e^{-ht} in the answer for $y(x,t)$ shows that when frictional resistance is introduced into the wave equation, the resulting solutions all decay in time, just like happens with real-life violin and guitar strings.
- Section 9.6 #13
- Iode investigation of the wave equation. Use the `Partial differential equations` module of Iode to solve the following two problems.
- Wave Problem D. Enter the wave equation with Dirichlet boundary conditions. Then enter $c = 2, L = 4, T = 4$, and for the intitial displacement $u(x, 0)$ enter `triangle(x,2,3)`, and for the initial velocity $u_t(x, 0)$ enter 0 (meaning release from rest). (*Note.* `triangle(x,a,b)` is an Iode function that has a triangular graph for $a < x < b$ and equals zero elsewhere.)

  The 3D-plot of the solution should show that as t increases, the triangle of height 1 at $t = 0$ splits into two overlapping triangles of height 1/2, one moving left and the other moving right. This is exactly as predicted by our work in class, where we used the the D'Alembert formula to derive the solution $u = F(x + ct) + F(x - ct)$ for release from rest.

  (i) Look at the snapshots of the solution, starting with $t = 0$. How far is the edge of the right-moving triangle from the endpoint at $x = 4$?

  Now step through the snapshots. How long does the edge of the triangle take to reach $x = 4$?

  Relate your answer to the interpretation of c as the wavespeed.

  (ii) In what way do the triangles change after they hit the endpoints? Illustrate with a snapshot, say from time $t = 1.5$.

  (iii) Explain how your conclusions are affected (if at all) when you re-do (i) and (ii) with initial displacement `bump(x,2,3)`, and with initial velocity still 0.

  Optional. You can also play around using `hat(x,2,3)` as the initial displacement, but you should first use the menu item `Change resolution` to increase the top harmonic, because the `hat` function has jumps and you need a lot of terms in its Fourier series to get a good approximation.
- Wave Problem N. Repeat Wave Problem D but with Neumann boundary conditions.