

NAME:

Math 285 Spring 2003 — Test 1

Total points: **75**. Do all questions. Explain all answers. No notes, books, calculators or computers.

1. [12=5+7 points] A flu epidemic hits Chicago. Write C for the total number of people in Chicago, and $N(t)$ for the number of people who are sick with the flu on the t^{th} day of the epidemic. Assume that each day, $\frac{1}{10}$ of the healthy people become sick and $\frac{1}{8}$ of the sick people become healthy again.

(a) Write down a differential equation involving $N(t)$. (Don't solve it.)

Note. Assume it is possible for a person to catch the flu more than once.

(b) What proportion of Chicagoans will be sick with the flu, after the epidemic has had time to spread thoroughly?

Hint. Phase line.

2. [4 points] Find an example of a function f for which the direction field of $\frac{dy}{dx} = f(x, y)$ shows a *vertically* repeating pattern. Explain.

3. [4 points] True/False, and Explain: if the direction field for $\frac{dy}{dx} = f(x, y)$ shows a *horizontally* repeating pattern, then each solution $y(x)$ must be periodic.

4. [20=14+2+4 points] Write $x(t)$ for the height at time t of an object that is falling downward under the influence of gravity. Assume the object encounters air resistance proportional to the square of its velocity $v(t) = x'(t)$. Then by Newton's Law we get

$$\frac{dv}{dt} = av^2 - g \quad (\text{for some positive constants } a \text{ and } g).$$

(a) Solve for $v(t)$. *Hint.* Write $b = \sqrt{g/a}$, to simplify the calculations.

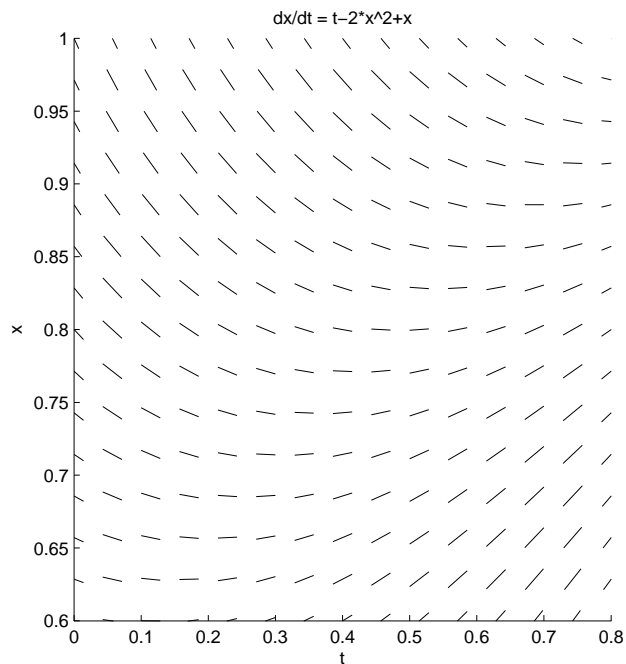
(b) Find the terminal velocity.

(c) Sketch the phase line for v . Does it agree with your answer to (b)?

5. [14 points]

Solve

$$(\sec^2 y)y' + (\sec^2 x) \tan y = e^{-\tan x}.$$



6. [15=9+2+4 points]

(a) “Halving the step size halves the error, in Euler’s method.” Illustrate the meaning of this principle with a suitable sketch above (take $(t_0, x_0) = (0, 1)$).

(b) State the Euler update formula.

(c) Consider $\frac{dx}{dt} = \sin(x - t^2)$, with $(t_0, x_0) = (0, 0)$ and $h = 0.1$. Evaluate

$$t_1 =$$

$$x_1 =$$

$$t_2 =$$

$$x_2 =$$

as accurately as you can.

7. [6=2+1+1+2 points] *No explanations are required, on this problem.*

(a) State the definitions of

$$\cosh(x) = \boxed{\phantom{e^x + e^{-x}}}$$

$$\sinh(x) = \boxed{\phantom{e^x - e^{-x}}}$$

(b) State the general solution of $y'' - 16y = 0$, in terms of cosh and sinh:

$$y = \boxed{}$$

(c) State the general solution of $y'' + 16y = 0$, in terms of cos and sin:

$$y = \boxed{}$$

(d) Solve $\frac{dw}{dx} = -2w$ with $w(1) = -1$. Sketch the solution.