

THE FIRST ORDER LINEAR METHOD

Call a Differential Equation **first order linear** if it can be put in the form

$$(1) \quad \frac{dy}{dx} + P(x)y = Q(x)$$

for some functions P and Q .

First Order Linear Solution Method.

Step 0. Put the equation into the standard form (1).

Step 1. Find the antiderivative $\int P(x) dx$ of P .

Note you don't need a constant of integration in this Step.

Step 2. Exponentiate it: $e^{\int P(x) dx}$.

Step 3. Multiply both sides of the Differential Equation by $e^{\int P(x) dx}$.

Step 4. Use the "reverse product rule" on the left side of the Differential Equation.

Step 5. Antidifferentiate both sides. Here you do need a constant of integration C .

Step 6. Rearrange to get a formula for y . Use the Initial Condition (if given) to evaluate C .

Example 1. Solve $y' + ay = b$.

Example 2. Solve

$$\frac{dx}{dt} + cx = a \cos(kt) + b \sin(kt).$$