MATH 442 — HOMEWORK 6

Section 3.3: 1

Temperature Underground: Solve the Temperature Underground problem from class by the following method. The point of this problem is to show that the "standard" method below is more difficult than the "intelligent guess" method used in class.

Method 2: Subtract off the boundary condition, then reflect, and then solve.

More precisely, let v be the temperature underground as studied in class, satisfying the heat equation $v_t = kv_{xx}$ with boundary condition $v(0,t) = \tau + \gamma\cos(\omega t)$. Define

$$V = v - v(0,t).$$

Find the boundary condition satisfied by V. Find the PDE satisfied by V. Assume V satisfies the initial condition

$$V(x,0) = \gamma e^{-Cx}\cos(Cx) - \gamma$$

where $C = \sqrt{\omega/2k}$. (This initial condition is chosen to make the problem work out nicely.) Then solve for V. Hence get v, and compare with the answer found in class.

Section 3.3: 3

Section 3.4: 2, 11. And for #11, interpret your answer in terms of a person wiggling one end of a stretched rope.

Additional Problem A: (Reduction of lower order terms in the Wave Equation) Consider the wave equation

$$u_{tt} - c^2 u_{xx} + \alpha u_t + \beta u_x + \gamma u = 0$$

for some constants α, β, γ. Make the change of variable

$$v(x,t) = \exp[(\alpha t - \beta c^{-2} x)/2] u(x,t)$$

and deduce the PDE satisfied by v. (You need not solve this PDE.)

Additional Problem B Suppose

$$v_{tt} - c^2 v_{xx} + \delta^2 v = 0, \qquad -\infty < x < \infty,$$

for some constant $\delta \in \mathbb{R}$. Write $w(x,y,t) = \cos(c^{-1}\delta y)v(x,t)$ and find a familiar PDE satisfied by w (it will involve x, y and t derivatives).

Additional Problem C Suppose

$$v_{tt} - c^2 v_{xx} - \delta^2 v = 0, \qquad -\infty < x < \infty.$$

Find a transformation that reduces to a PDE in x, y, t, similar to Problem B.

Conclusions. Additional Problems A, B and C show how the one dimensional wave equation with lower order terms can always be reduced to the two dimensional wave equation. (Section 9.2 in the textbook shows how to solve the two dimensional wave equation.)