

MATH 442 — HOMEWORK 7

Section 4.1: 1

Section 4.1: 2 (Strauss means Dirichlet BCs here, although the physical situation corresponds to Robin BCs)

Section 4.1: 4 (explain where the assumption $0 < r < 2\pi c/l$ is used)

Section 4.2: 2 (for (a), it is better to write the eigenfunctions as $\cos(n\pi x/2l)$ for odd integers n)

Section 4.3: 1, under the assumption $a < -1/l$. (You don't need to read Section 4.3 to do this problem; but note you are trying to get a formula involving the tangent or hyperbolic tangent function, like (10), and then sketch graphs to find intersection points.)

Additional Problem A: Eigenvalues and eigenfunctions for periodic BCs. Find all eigenfunctions and eigenvalues for $X'' = -\lambda X$ on the interval $-\ell < x < \ell$, under the periodic boundary conditions

$$X(-\ell) = X(\ell), \quad X'(-\ell) = X'(\ell).$$

Be sure to consider the cases $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$.

Additional Problem B: Wave equation with periodic BCs.

(a) Solve the wave equation on $-\ell < x < \ell$ with Periodic BC and with IC $u(x, 0) = \phi(x)$, $u_t(x, 0) = \psi(x)$. (Assume ϕ and ψ can be expanded in full Fourier series.) [This problem is taken from the Day 22 Worksheet.]

(b) Sketch a standing wave at several times.

(c) Sketch a traveling wave at several times.