

Math 442 — Day 11 Worksheet

Material: Section 2.4 – Diffusion $u_t = ku_{xx}$ on the whole line

Exercise 1. Meaning of the Source Function Write a short paragraph to justify the following statement:

Let y_0 be fixed. Then the function $S(x - y_0, t)$ gives the chemical concentration at position x and time t that results from a unit mass of chemical being released at position y_0 at time 0.

Exercise 2. Erf function Define

$$\operatorname{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2} dz.$$

(It is called the *error function* due to its applications in probability theory.)

Show $Q(x, t) = \frac{1}{2} + \frac{1}{2}\operatorname{Erf}(x/\sqrt{4kt})$.

Exercise 3. Erf function Solve the diffusion equation $u_t = ku_{xx}$ for $-\infty < x < \infty$, with initial condition

$$u(x, 0) = \begin{cases} 1 & \text{when } |x| < L, \\ 0 & \text{otherwise.} \end{cases}$$

Here $L > 0$ is a constant. You can express your answer in terms of Erf.

[*Hint.* First express the initial condition as a difference of step functions.]

Also, sketch the solution for $t = 0, 1, \infty$.