

Day 40 Worksheet

Periodic function, $-l < x < l \Rightarrow$ use Fourier series, $c_k = \text{amplitude at } k\text{-th frequency}$

Non-periodic function, $-\infty < x < \infty \Rightarrow$ use Fourier transform, $F(k) = \text{amplitude at freq. } k$

① Verify the transforms (5), (6) on p. 326.

② Note: ⊙ transform (10) is just (5) in reverse (put $F(k) = 2\pi S(k)$ into (3))
⊙ transform (11) is done on p. 326

③ Verify properties (i) - (vi) on p. 327. Hint for (ii): substitute $i \frac{dF}{dk}$ into (3) in place of "F".

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|---|-------------------------------------|
| Property (i) says the Fourier transform takes | differentiation to multiplication |
| " (ii) | " multiplication to differentiation |
| " (iii) | " translation to modulation |
| " (iv) | " modulation to translation |
| " (vi) | " dilation "out" to dilation "in" |

And Property (v) says the Fourier transform is a linear operation.

Space for calculations:

L40-2

④

Fix $t > 0$, $\tilde{k} > 0$.

Let $f(x) = S(x, t)$, the source fn. for diffusion eq. with diffusivity \tilde{k} .

$$= \frac{1}{\sqrt{4\pi\tilde{k}t}} e^{-x^2/4\tilde{k}t}$$

Different letter from Fourier transform variable!

Find the Fourier transform.

HINT: $f(x) = g\left(\frac{1}{\sqrt{2\tilde{k}t}}x\right) / \sqrt{4\pi\tilde{k}t}$

where $g(x) = e^{-x^2/2}$.