Math 442 — Day 42 Worksheet

Material: Section 12.4 – Solving linear, constant coeff. PDEs by transforms.

$$U(k,t) = \int_{-\infty}^{\infty} u(x,t)e^{-ikx}\,dx$$

is the Fourier transform of $u(x,t)$ with respect to the x-variable.

Exercise 1. Diffusion equation by Fourier transform

Take the Fourier transform of the diffusion equation

$$u_t = \widetilde{k}u_{xx}, \qquad -\infty < x < \infty, \quad t > 0,$$

and also transform the initial condition $u(x,0) = f(x)$. Solve the resulting ODE in the t-variable. Then invert the Fourier transform to obtain $u(x,t)$.

Exercise 2. Wave equation by Fourier transform

Take the Fourier transform of the wave equation with lower order terms

$$u_{tt} = c^2 u_{xx} + \alpha u_x + \beta u_t + \gamma u, \qquad -\infty < x < \infty, \quad t > 0,$$

and also transform the initial conditions $u(x,0) = f(x), u_t(x,0) = g(x)$. Solve the resulting ODE in the t-variable, assuming $\alpha = \beta = \gamma = 0$. Then invert the Fourier transform to obtain the D'Alembert formula for $u(x,t)$.