Math 545 Homework 3

Due: Tuesday 2 December by 6pm, to my office (376 Altgeld Hall)

Problem 1 (Adjoint of Fourier transform)**.**
Find the adjoint of the Fourier transform on $L^2(\mathbb{R}^d)$.

Problem 2 (Periodization connects Fourier series and transforms)**.**
Suppose $f \in L^1(\mathbb{R}^d)$.

(a) Prove that the periodization

$$\mathrm{Pe}(f)(x) = (2\pi)^d \sum_{n \in \mathbb{Z}^d} f(x + 2\pi n)$$

of f is $2\pi\mathbb{Z}^d$-periodic and belongs to $L^1(\mathbb{T}^d)$, with

$$\|Pe(f)\|_{L^1(\mathbb{T}^d)} \leq \|f\|_{L^1(\mathbb{R}^d)}.$$

(b) Deduce from your argument that the series for $\mathrm{Pe}(f)(x)$ converges pointwise absolutely, for almost every x.

(c) Show that the jth Fourier coefficient of $\mathrm{Pe}(f)$ equals the Fourier transform of f at j:

$$\widehat{\mathrm{Pe}(f)}(j) = \widehat{f}(j), \qquad j \in \mathbb{Z}^d$$

Problem 3 (Course summary)**.**
Write a one page description of the most important and memorable results and general techniques from this course. Be brief, but thoughtful; explain how these main results fit together.

You need not state the results technically — intuition is more helpful than rigor, at this stage.

Note. If your lecture notes are incomplete, then check the online notes at

www.math.uiuc.edu/~laugesen/545/topics.html

The notes cover most of the course.