Math 553 — Fall 2007 — Homework 11

Due: Monday 3 December, by noon.

(1) McOwen 5.2.7

(2) McOwen 5.3.3

(3) McOwen 5.3.4

(4) McOwen 5.3.5

(5) *(Critical mass for uranium.)* The neutron density $n(x,t)$ inside a lump of Uranium-235 obeys the differential equation

$$n_t = k\Delta n + cn$$

for some positive constant c. (Here c represents the rate at which neutrons in Uranium-235 collide with nuclei and hence cause further neutrons to be released by nuclear fission.) Assume the lump is spherical with radius R.

Find the critical radius R_0 such that if $R > R_0$ then n will increase exponentially with time. (Assume n is radial, *i.e.* depends only on r, with boundary condition $n = 0$ when $r = R$, on the surface of the ball.)

Conclusion. To prevent accidental explosions, you should only use lumps with radius $R < R_0$, in the laboratory.