

1. Suppose z is given implicitly as a function of x and y by the equation

$$x^3 z + y \cos z + \frac{\sin y}{z} = 0.$$

Find $\frac{\partial z}{\partial x}$.

2. Let S be the set of points in \mathbb{R}^3 described by the equation

$$\sin(xy) + e^{xz} + x^3 y = 1.$$

At what points of S can z be described as a function of x and y ?

3. What is the second Taylor polynomial of $f(x, y) = e^{2x+y}$ at $(0, 0)$?

4. Show that the surfaces $z = 7x^2 - 12x - 5y^2$ and $xyz^2 = 2$ intersect orthogonally at $(2, 1, -1)$.

5. For what values of the parameter k does the function

$$f(x, y, z) = kx^2 + kxz - 2yz - y^2 + \frac{k}{2}z^2$$

have a nondegenerate local maximum at $(0, 0, 0)$? What about a nondegenerate local minimum?

6. Find the point on the intersections of the two planes $x - 2y + 3z = 8$ and $2z - y = 3$ which is closest to the point $(2, 5, -1)$.

7. $\int_0^1 \int_{3y}^3 \cos(x^2) dx dy = ?$

8. Compute $\iint_D (2x + y)^2 e^{x-y} dA$ where D is the region enclosed by $2x + y = 1$, $2x + y = 4$, $x - y = -1$ and $x - y = 1$.

9. Is the vector field $\vec{F} = (e^x \sin y, e^x \cos y, 3z^2 + 2)$ conservative? If it is, find a potential for \vec{F} .

10. Let $\alpha = M(x, y) dx + N(x, y) dy$ be a 1-form in $\mathbb{R}^2 \setminus \{(0, 0)\}$.

If $d\alpha = 0$, is it true that $\int_C \alpha = 0$ for any simple closed curve C ? Explain.

Is $\int_C \alpha = 0$ if $d\alpha = 0$ and C does not enclose the origin? Explain.

11. Compute $\int_C xyz ds$ where C is the line segment in \mathbb{R}^3 with end points $(0, 0, 0)$ and $(2, 4, 6)$.

12. Show that for any simple closed curve C

$$\int_C (-y^3 dx + (x^3 + 2x + \sin(y^2)) dy) \neq 0.$$

13. Compute $\iint_{\Sigma} (x^2 + y^2 + z^2) dS$ where $\Sigma = \{(x, y, z) \mid x^2 + y^2 + z^2 = a^2\}$

14. Compute $\int_{\Sigma} z \, dS$ where $\Sigma = \{(x, y, z) \mid x^2 + y^2 = 1, 0 \leq z^2 \leq 5\}$

15. Let S_r denote the sphere in \mathbb{R}^3 of radius r centered at the origin and oriented with outward normal. Suppose \vec{F} is a vector field on \mathbb{R}^3 such that $\int_{S_r} \vec{F} \cdot dS = 2r + 6$. Compute

$$\iiint_D \nabla \cdot \vec{F} \, dV,$$

where $D = \{(x, y, z) \mid a^2 \leq x^2 + y^2 + z^2 \leq b^2\}$ for some constants $a, b > 0$ with $a^2 < b^2$.

16. Is $\gamma(t) = (\cos(t), \sin(t))$ a flow line of the vector field $\vec{F}(x, y) = (y, -x)$?

17. Let $\Sigma = \{(x, y, z) \mid z = e^{1-x^2-y^2}, z \geq 1\}$ be oriented with upward normal. Compute

$$\iint_{\Sigma} (x, y, 2 - 2z) \cdot d\vec{S}.$$

18. Is the parameterization

$$X : \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\} \rightarrow \mathbb{R}^4, \quad X(x, y, z) = (x, y, z, x^2 + y^2 + z^2)$$

of a 3-manifold M in \mathbb{R}^4 compatible with the orientation form $\Omega = xdy \wedge dz \wedge dw - ydx \wedge dz \wedge dw + zdx \wedge dy \wedge dw - wdx \wedge dy \wedge dz$?

Compute $\int_M dx \wedge dy \wedge dz$ where M is oriented by Ω .

19. Compute

$$\int_{S^4} x_3 dx_1 \wedge dx_2 \wedge dx_4 \wedge dx_5 + x_4 dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_5$$

where $S^4 = \{(x_1, x_2, x_3, x_4, x_5) \mid x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 5\}$ is oriented as the boundary of the region $D = \{(x_1, x_2, x_3, x_4, x_5) \mid x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 \leq 5\}$.

20. Let $\Sigma \subset \mathbb{R}^3$ be a surface and $\vec{F} = (x^2y, -z^3, \cos x)$ a vector field. Find a 2-form ω so that

$$\iint_{\Sigma} \vec{F} \cdot d\vec{S} = \iint_{\Sigma} \omega$$

21. Let C be a curve in \mathbb{R}^4 . Find a 1-form α so that

$$\int_C (\cos x, \sin y, z \sin w, w^2) \cdot d\vec{s} = \int_C \alpha.$$