

Homework #4, Math 241 FD1 and FD2 Due Wednesday, 9/22/09

Read 2.2, 2.3, 2.4, 2.5, 2.6.

Problems: 2.4 # 19b, 20a.

2.5 #11, 15, 20, 21, 26, 27

2.6 #1, 2, 3, 9

Additional problems.

1 The matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{pmatrix}$ defines a function $F : \mathbb{R}^4 \rightarrow \mathbb{R}^2$. Write down a formula for $F(x, y, z, w)$. Is F linear?

2 Using the ε - δ definition of continuity prove that the function $f(x, y) = 2x + 3y - 7$ is continuous at $(a, b) \in \mathbb{R}^2$.

Hints: (i) $|x - a| \leq \|(x - a, y - b)\|$. (ii) $\delta = \epsilon/6$ may work.

3a Compute the partial derivatives $\frac{\partial f}{\partial x}(0, 0)$, $\frac{\partial f}{\partial y}(0, 0)$ of the function

$$f(x, y) = \begin{cases} \frac{x^3 + y^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases} .$$

3b Prove that the function $f(x, y)$ in **3a** is differentiable at $(0, 0)$.

3c Is the function $g(x, y) = \frac{\partial f}{\partial x}(x, y)$ continuous at $(0, 0)$? Prove your answer.