worksheet 2, Math 241 (for the second midterm)

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1 True/false questions (unless noted otherwise) .

a. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function with $f(a) = 2$ and $\nabla f(a) \neq 0$. Then $\nabla f(a)$ is perpendicular at a to the hypersurface

$$S = \{x \in \mathbb{R}^n \mid f(x) = 2\}.$$

b. If $\nabla f(a) = 0$ then f has either a local maximum or a local minimum at the point a.

c. Suppose $g : \mathbb{R}^n \to \mathbb{R}$ is a function. Suppose further the restriction of a function $f : \mathbb{R}^n \to \mathbb{R}$ to the constraint $S = \{x \in \mathbb{R}^n \mid g(x) = 2\}$ achieves its maximum at a point $x_0 \in S$. Which of the following three statements are true (assume ∇g does not vanish on S)?

1. $\nabla f(x_0) = 0$
2. $\nabla f(x_0)$ is perpendicular to S at x_0.
3. $\nabla f(x_0)$ and $\nabla g(x_0)$ are proportional.

d.

$$\int_0^2 \int_0^y f(x,y)\,dx\,dy = \int_0^2 \int_0^x f(x,y)\,dy\,dx$$

2 Suppose the function $f(x,y)$ satisfies

$$f(P) = 10, \quad \nabla f(P) = 0, \quad \frac{\partial^2}{\partial x^2} f(P) = 1, \quad \frac{\partial^2}{\partial x \partial y} f(P) = 2, \quad \frac{\partial^2}{\partial y^2} f(P) = 2.$$

Then

1. P is a local maximum of f
2. P is a local minimum of f
3. P is a saddle point of f
4. P is a critical point but not one of the three kinds above
5. P is not a critical point of f
6. there is not enough data to determine what kind of a point P is.

3. Find the point on the intersections of the two planes $x - 2y + 3z = 8$ and $2z - y = 3$ which is closest to the point $(2, 5, -1)$.

4. $\int_0^1 \int_{3y}^3 \cos(x^2)\,dx\,dy =$

5 $\int_0^1 \int_x^1 x^2\,dy\,dx =$

6 $\int_0^2 \int_{y/2}^1 ye^{x^3}\,dx\,dy =$

7 Consider the homogeneous solid region of density 1 defined in spherical coordinates by

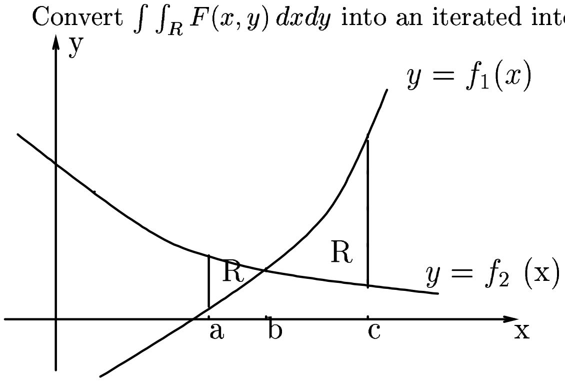
$$0 \leq \rho \leq 1 \quad \pi/6 \leq \varphi \leq \pi/3$$

a Sketch the region.
b Write the z coordinate $\bar{z}$ of the center of mass as the ratio of two triple integrals in spherical coordinates. DO NOT EVALUATE THE INTEGRALS.
c What are the x and y coordinates of the center of mass? Explain.

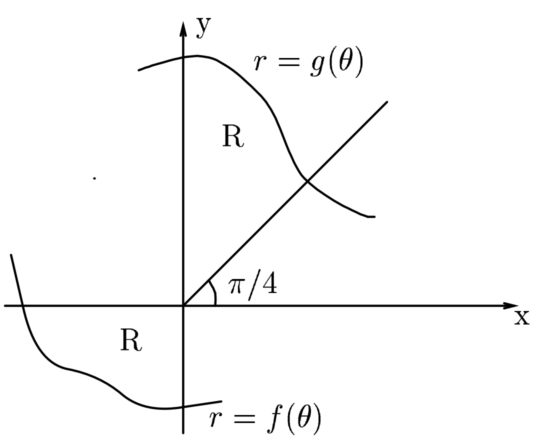
8 Consider the solid defined by the inequalities $0 \leq z \leq 3$ and $x^2 + y^2 \leq z^2$. Suppose the density of the solid is given by $\delta(x, y, z) = 2z$. What is the mass of the solid?

9 Classify the critical points of the function $f(x, y) = 6xy^2 - 2x^3 - 3y^4$ (you may wish to use the second derivative test)

10 Convert $\int\int_R F(x, y)\, dxdy$ into an iterated integral, where the region R is pictured below.



12 Write down the iterated integral that computes the area of the region R pictured below.



13 Describe the object defined in spherical coordinates (ρ, ϕ, θ) by the given equation in a few words.
a) $\phi = \pi/2$
b) $\phi = 0$
c) $\phi = 2\pi/3$
d) $\rho = 5$
e) $\theta = 0$
f) $\rho = \cos\phi$
g) $\rho\cos\phi = 1$
h) $\rho\sin\phi = 1$