347 Review

Note Title

The following is a list of topics we have covered.

- rational and irrational numbers.
- upper / lower bounds, completeness axiom for $\mathbb{R}$.
- geometric series, decimal expansion, pigeonhole principle, periodic / nonperiodic decimals.
- axioms for a field, ordered field, complete ordered field, consequences of the axioms.
- $\mathbb{C}$, $e^{ix} = \cos x + i \sin x$
- triangle inequality for $\mathbb{R}$ and $\mathbb{C}$
- induction
- well-ordering, well-ordering $\Rightarrow$ induction principle
- sequences in $\mathbb{R}$, limits of sequences
- $\sup S$ as a limit, limits are unique
- bounded sequences, convergent $\not\Rightarrow$ bounded
  limit theorems: sum, product, ratio...
- $\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$ for $|q|<1$
- $(a_n \to a,\ b_n \to b,\ a_n \le b_n) \Rightarrow a \le b$.
- $a_n > 0$, $a_{n+1}/a_n \to L$, $L<1 \Rightarrow a_n \to 0$
- monotone sequences and their convergence
- Cauchy sequences, squeezing theorem
- Cauchy $\Leftrightarrow$ convergent
- subsequences, limits of subsequences of convergent sequences
- Bolzano-Weirstrass for sequences in $\mathbb{R}$ and $\mathbb{C}$.

- Continuity (ε-δ definition), continuity and convergent sequences.
- $f:[a,b] \to \mathbb{R}$ is continuous $\Rightarrow$ f is bounded. In particular f assumes its max and min on $[a,b]$.
- Intermediate value theorem; existence of $\sqrt[n]{x}$.
- functions, composition of functions
- injective, surjective, bijective, invertibility.
- countable, uncountable, finite
- $f: \mathbb{N} \to A$ onto $\Rightarrow$ A is countable
- infinite subsets of $\mathbb{N}$ are countable
- $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}$ are countable, countable union of countable sets is countable
- $[0,1]$ is not countable. No bijection $X \to \mathcal{P}(X) = 2^X$.
- divisibility in $\mathbb{Z}$, division algorithm in $\mathbb{Z}$ from well-ordering existence and uniqueness of gcd

  $\gcd(a,b) = ua + vb$ for some $u, v \in \mathbb{Z}$
- primes; p prime, $p \mid ab \Rightarrow p \mid a$ or $p \mid b$. Existence and uniqueness of factorization into primes
- mod n arithmetic
- equivalence relations, equivalence classes
- rings, $\mathbb{Z}_n$, $\mathbb{R}[x]$
- division algorithm in $\mathbb{R}[x]$
- $\mathbb{R}[x]/(x^2+1) \simeq \mathbb{C}$.