Homework 5, Math 347, **Prof. Eugene Lerman**
Due Friday, February 22, 2008 (in class)

This of this homework as a (partial) preparation for the midterm.

1 Let $X = \mathbb{R}^2$ and $\sim$ the relation defined by

$$(a, b) \sim (a', b') \Leftrightarrow a^2 - b = (a')^2 - b'$$

a) Check that this is an equivalence relation.
b) Describe the equivalence classes of $(1, 1)$, $(2, 4)$ and of $(2, 2)$ as subsets of $\mathbb{R}^2$.

2 Is the set J of polynomials of degree at most 1 an ideal in $\mathbb{R}[x]$? Note that

$$J = \{a + bx \mid a, b \in \mathbb{R}\}.$$

Prove your answer.

3 (a) Show that the partition of $\mathbb{R}$ into the 3 sets

$$\{0\}, (-\infty, 0), (0, \infty)$$

corresponds precisely to the equivalence relation

$$x \sim y \Leftrightarrow x = ty \text{ for some } t > 0.$$

That is, the equivalence classes of $\sim$ are precisely the three sets above.

(b) Find $x, y, z, w \in \mathbb{R}$ so that $x \sim y$, $z \sim w$ but $x + z$ is not equivalent to $y + w$.