Worksheet 2, Math 347, **Prof. Eugene Lerman**
Friday, February 15, 2008

1 Which of the following relations are equivalence relations? Justify your answer. That is, check the three conditions.

a. Fix an integer $n \neq 0$. Define a relation $\sim$ on $\mathbb{Z}$ by $x \sim y$ if and only if $n|(x-y)$.

b Define a relation $\sim$ on $\mathbb{Z}$ by $a \sim b$ if and only if $a|b$.

c Define a relation $\sim$ on $\mathbb{R}^2$ by $(a,b) \sim (a',b')$ if and only if $a+b = a'+b'$.

d Define a relation $\sim$ on $\mathbb{R}^2$ by $(a,b) \sim (a',b')$ if and only if $2a^2+b = 2(a')^2+b'$.

e Define a relation $\sim$ on $\mathbb{R}^2$ by $(a,b) \sim (a',b')$ if and only if $a = a'$.

f Fix a polynomial $d(x)$. Define a relation $\sim$ on $\mathbb{R}[x]$ by $f \sim g$ if and only if $d|(f-g)$.

2 Recall that given an equivalence relation $\sim$ on a set X, the **equivalence class** of $x \in X$ is the set

$$[x] = \{y \in X | x \sim y\}.$$

a Let $X = \mathbb{R}^2$ and $\sim$ the equivalence relation defined by $(a,b) \sim (a',b')$ if and only if $b = b'$. What is the equivalence class of $(1,1)$? Draw a picture of the class.

b Let $X = \mathbb{Z}$ and $\sim$ the equivalence relation defined by $a \sim b$ if and only if $3|(b-a)$. What are the equivalence classes of 1, 10, 11 and 12? Can it happen that $a \neq b$ and $[a] = [b]$?

c Let $X = \mathbb{R}^2$ and $\sim$ the relation defined by $(a,b) \sim (x,y)$ if and only if $a^2+b^2 = x^2+y^2$. What is the equivalence class of $(1,0)$? Draw a picture of the class.

3 Let $J = \{3x+2 \mid x \in \mathbb{Z}\}$. Is J an ideal of $\mathbb{Z}$? Explain.