worksheet # 1, Math 380 (for the midterm)
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True/false questions, unless noted otherwise.

1 The map $f(x, y) = (x + y - 2, x - y)$ is linear.

2. If $F(x_1, \ldots, x_n) = (F(x_1, \ldots, x_n), \ldots, F_m((x_1, \ldots, x_n))$ is linear, then

$$DF(a)h = F(h)$$

for all $a, h \in \mathbb{R}^n$.

3. The second Taylor polynomial of $f(x, y) = e^{x+y}$ at $(0, 0)$ is $1 + (x + y) + \frac{1}{2}(x + y)^2$.

4. The second Taylor polynomial of $f(x, y) = e^{x+y}$ at $(1, 2)$ is $1 + ((x - 1) + (y - 2)) + \frac{1}{2}((x - 1) + (y - 2))^2$.

5. The level set of a function $f(x, y, z)$ is either empty or a surface.

6. If $\lim_{\vec{x} \to \vec{0}} f(\vec{x})$ exists and is finite, then f is continuous at 0.

7. If a function $f(x, y)$ has both partials at (a, b) then f is differentiable at (a, b).

8. The directional derivative of $f(x, y, z)$ in the direction of $(0, 0, 1)$ is $\frac{\partial f}{\partial z}$.

9. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function with $f(a) = 2$ and $\nabla f(a) \neq 0$. Then $\nabla f(a)$ is perpendicular at a to the hypersurface

$$S = \{x \in \mathbb{R}^n \mid f(x) = 2\}.$$

10. Assume $f(1, 2) = 3$. If $\frac{\partial f}{\partial x}(1, 2) \neq 0$ then the equation

$$f(x, y) = 3$$

implicitly defines y as a function of x near $x = 1$.

11. The divergence of a vector field is a vector field.

12. If $f : \mathbb{R}^n \to \mathbb{R}$ is a function and $\gamma : \mathbb{R} \to \mathbb{R}^n$ a curve, then

$$\frac{d}{dt}(f(\gamma(t))) = \nabla f(\gamma(t)) \cdot \gamma'(t).$$

13. Suppose $F, G : \mathbb{R}^n \to \mathbb{R}^n$ are two functions such that $F(G(x)) = x$ for all $x \in \mathbb{R}^n$. Then

$$DF(G(x))DG(x)$$

is the identity matrix for any $x \in \mathbb{R}^n$.

14. For any function $f : \mathbb{R}^n \to \mathbb{R}$

$$div(\nabla f) = 0.$$

15. For any vector field $\vec{F}$ on $\mathbb{R}^3$

$$div(curl\,\vec{F}) = 0.$$

16. $\gamma(t) = (\cos(t), \sin(t))$ is a flow line of the vector field $\vec{F}(x, y) = (x, -y)$.

17. If $\nabla f(a) = 0$ then f has either a local maximum or a local minimum at the point a.

18. If the Hessian of a function f at a is positive definite, then f has a local minumum at the point a.

19. The set $D = \{(x, y, z) \in \mathbb{R}^3 \mid 0 \leq x \leq 1, y \leq 2, 2 \leq z \leq 3\}$ is compact.

20. Let D be a region in $\mathbb{R}^3$ as above. Then every continuous function $f : D \to \mathbb{R}$ achieves its maximum at some point of D.

21. Suppose $g : \mathbb{R}^n \to \mathbb{R}$ is a function and $S = \{x \in \mathbb{R}^n \mid g(x) = 2\}$ is a hypersurface. Suppose further the restriction of a function $f : \mathbb{R}^n \to \mathbb{R}$ to S achieves its maximum at a point $x_0 \in S$. Which of the following three statements are true?

1. $\nabla f(x_0) = 0$
2. $\nabla f(x_0)$ is perpendicular to S at x_0.
3. $\nabla f(x_0)$ and $\nabla g(x_0)$ are proportional.

22. If $T(u, v) = (2u + v, u + 3v)$ then the area of the image $T(R^*)$ of the unit square $R^* = [0, 1] \times [0, 1]$ is 5.

23.

$$\int_0^2 \int_0^y f(x, y)\, dx\, dy = \int_0^2 \int_0^x f(x, y)\, dy\, dx$$