Set 1 of problem bank questions, Math 425, Prof. Eugene Lerman (Do not turn in. May show up on a midterm.)

1 Suppose $A : \mathbb{R}^n \to \mathbb{R}^m$ is a linear map and $b \in \mathbb{R}^m$ is a vector. Show that $F : \mathbb{R}^n \to \mathbb{R}^m$, $F(x) = Ax + b$ is differentiable at every point of $\mathbb{R}^n$ and that $DF(a) = A$ for all $a \in \mathbb{R}^n$.

2 Consider $\mathbb{R}^{n^2}$ as a vector space of $n \times n$ matricies. Show that the function $F : \mathbb{R}^{n^2} \to \mathbb{R}^{n^2}$, $F(A) = AA$ (matrix multiplication) is differentiable at all points $A \in \mathbb{R}^{n^2}$ and that $DF(A)H = AH + HA$ for all $H \in \mathbb{R}^{n^2}$.

3 Let $B : \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}^m$ be a bilinear map. Prove that B is differentiable at every point $(a, b) \in \mathbb{R}^n \times \mathbb{R}^k$ and that

$$DB(a, b)(h, k) = B(a, k) + B(h, b)$$

for all $(h, k) \in \mathbb{R}^n \times \mathbb{R}^k$.