

1 Let Q be a rectangle in \mathbb{R}^n

(a) Prove that if a bounded function $f : Q \rightarrow \mathbb{R}$ is 0 except possibly on a set of measure 0, then f is integrable and $\int_Q f = 0$. Hint: proof of 3.12 in notes may be useful.

(b) Suppose $f : Q \rightarrow \mathbb{R}$ is integrable and $g : Q \rightarrow \mathbb{R}$ is a bounded function that equals f everywhere except possibly on a set of measure 0. Prove that g is integrable and that $\int_Q f = \int_Q g$.

2 Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a C^2 function. Give an alternative proof that mixed partials commute by using Fubini's and the fundamental theorem of calculus. That is, compute $\int_Q \frac{\partial^2 f}{\partial x \partial y} - \int_Q \frac{\partial^2 f}{\partial y \partial x}$.

3 Let $S \subset \mathbb{R}^n$ be a rectifiable set and $f : S \rightarrow \mathbb{R}$ (bounded and) integrable. Prove that

$$\left| \int_S f \right| \leq \left(\sup_{x \in S} |f(x)| \right) \text{vol}(S).$$