Homework #5 Due Monday, 2/25, 10am

Note Title

#1 Let $A \subseteq \mathbb{R}^n$ be an open set, $f, g : A \to [0, \infty)$ continuous functions with $f \leq g$. Prove: if $\int_A^{\#} g$ exists, then so does $\int_A^{\#} f$.

#2 a) Prove that $\int_{[0,+\infty)}^{\#} \frac{1}{x^2}$ exists.

b) Prove that $\int_{[0,+\infty)}^{\#} \frac{\sin x}{x^2}$ exists.

#3 Prove that $\lim_{R \to +\infty} \int_{[-R,R]} \frac{1+x}{1+x^2}$ exists but $\int_{\mathbb{R}}^{\#} \frac{1+x}{1+x^2}$ does not.

#4 Let $f(x,y) = \frac{1}{(y+1)^2}$. Let A and B be the open sets

$$A = \{ (x,y) \mid x > 0 \text{ and } x < y < 2x \}$$

$$B = \{ (x,y) \mid x > 0 \text{ and } x^2 < y < 2x^2 \}.$$

Prove that $\int_A^{\#} f$ does not exist; show that $\int_B f$ does exist and calculate it.

Hint: Use Theorem 14.4 on p 116 of Munkres.