1. Homework due Tuesday, August 29, 9 a.m.

Section 1.1: #12, 14, 26, 28, and 36.

Section 1.2: #4, 6, 10, 20, and 26.

2. Homework due Thursday, August 31 at 9 a.m.

Section 1.3: #4, 8, 10, 12, 14, and 16.

Section 1.4: #14, 16, 22, and 24.

This homework has now been fixed. If you worked it before by guessing, try it again to get the idea. I will take the higher of your scores on the broken homework and this fixed homework.

3. More on Notes

The notes on the class webpage should help you with the lectures, and the lectures will be necessary for most students as an aid in following the notes. I will put a draft of the next lecture on the class web-site. The notes may change from the draft and will go beyond the lecture for that day.

4. More on Absolute Value

The important facts to remember about the absolute value are the following: For any a and b, $|ab| = |a|\,|b|$, in particular $|-a| = |a|$, so $|a-b| = |b-a|$, and we also have the triangle inequlity $|a+b| \le |a| + |b|$.

An equation like $3 \le |x|$ is satisfied by all $x \ge 3$ along with all $x \le -3$. You should **NOT** write that the inequality is satisfied by all x with $3 \le x \le -3$, because this says that $3 \le -3$.

5. Combinations of Functions

Let f and g be functions of x, and let c be a constant.

1) On the domain of f, we can form the function cf by setting $cf(x) = c \cdot f(x)$.

2) On the intersection of the domains of f and of g, (i.e., the set of points in both domains) we can form $f+g$, $f-g$, and $f \cdot g$ by setting $(f+g)(x) = f(x)+g(x)$, $(f-g)(x) = f(x) - g(x)$, and $(f \cdot g)(x) = f(x) \cdot g(x)$.

3) On the intersection of the domains of f and of g, (i.e., the set of points in both domains) where $g(x) \neq 0$, we can form f/g by setting $(f/g)(x) = f(x)/g(x)$.

4) On the set of points x in the domain of f such that $f(x)$ is in the domain of g, we can form the composite function $x \mapsto g(f(x))$. Other notation for a composite function is $g \circ f$. That is, $g \circ f(x) = g(f(x))$.

EXAMPLES: $f(x) = \sqrt{x}$, $g(x) = 3x - 1$, $c = \pi$.

1) $cf(x) = \pi\sqrt{x}$ for $x \geq 0$.

2) $(f+g)(x) = \sqrt{x}+3x-1$, $(f-g)(x) = \sqrt{x}-3x+1$, and $(f \cdot g)(x) = \sqrt{x}\,(3x-1)$ for $x \geq 0$.

3) $(f/g)(x) = \frac{\sqrt{x}}{3x-1}$ for $0 \leq x < \frac{1}{3}$, and $\frac{1}{3} < x$.

4) $g(f(x)) = 3\sqrt{x} - 1$ for $x \geq 0$. On the other hand, $f(g(x)) = \sqrt{3x-1}$, for $x \geq \frac{1}{3}$.

6. QUADRATIC EQUATIONS

For the plane, after linear equations of the form $Ax + By + C = 0$, with not both A and $B = 0$, the next "level up" are the quadratic equations. In general, these have the form

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

with at least A or B or C not equal to 0. The solution set of such an equation is the set of points whose coordinates satisfy the equation. In general, the solution sets for these equations are not the graphs of functions. That is, except for special cases, you must pick a part of the plot of the solution set to get the graph of a function.

A simple example is the plot of a circle with center $(0,0)$ and radius $r > 0$. This is given by the formula $x^2 + y^2 = r^2$. An equivalent form is $y = \pm\sqrt{r^2 - x^2}$. To get y as a function of x, you must choose either the $+$ or the $-$, that is choose the top of the circle or the bottom, and restrict the value of x to the interval $[-r, r]$. A similar statement is true for circles with center (x_0, y_0); the equation here is

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

Here we have a simple example of translating a curve so that the origin goes to the point (x_0, y_0). That is, if you replace x with $x - x_0$ and y with $y - y_0$, then the point (x_0, y_0) acts like the origin.

For our course, the mixed xy term will usually not appear. If the coefficients of both x^2 and y^2 are the same, we can expect a circle, or a single point or no points at all. For example, if we have $x^2 + y^2 + 4x + 6y + a = 0$, then we complete the square twice to get

$$\begin{aligned}(x+2)^2 + (y+3)^2 - 4 - 9 + a &= 0,\\ (x+2)^2 + (y+3)^2 &= 13 - a.\end{aligned}$$

If $a < 13$, we have a circle, If $a = 13$, we have a single point, otherwise the solution set is empty.

If our quadratic equation has the form

$$Ax^2 + Dx + Ey + F = 0,$$

with $A \neq 0$ and $E \neq 0$, then after dividing by A and doing some algebra, we have the equation

$$y = ax^2 + bx + c.$$

This is the equation of a parabola; it determines y as a function of x. The domain of this function is the whole real line. To see what is going on, you complete the square.

The y-coordinate of the vertex of a parabola is the minimum value of the function if the parabola opens up; it is the maximum value if the parabola opens down. Notice that the tangent line to the parabola is horizontal at the vertex. This give the clue as to how we will find points where functions take maximum and minimum values using calculus.

In this course, we will be working with parabolas where the axis of symmetry is parallel to the x-axis or, sometimes, the y-axis. There are, however, other parabolas; they are given by quadratic equations with mixed (i.e., nonzero xy) terms.

A good problem to practice completing the square is to derive the quadratic formula starting from the equation $ax^2 + bx + c = 0$ with $a > 0$. One writes the equivalent equation $x^2 + \frac{b}{a}x = -\frac{c}{a}$, completes the square, to get $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$, and then takes the positive and negative square root to isolate the value of x.

7. Polynomial Functions

Let n be a positive natural number; i.e., $n = 1$ or 2 or 3, etc. A polynomial of degree n has the form

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where the a's are real numbers and $a_n \neq 0$. The domain of such a function is the real line, unless it is restricted to a subset. If $n = 1$, we have the equation of a straight line $y = p(x) = a_1 x + a_0$. If $n = 2$, we have the equation of a parabola.

There are **at most** n points x on the real line where an nth degree polynomial takes the value 0; i.e., where $p(x) = 0$.

EXAMPLES: $p(x) = x^2 + 1$ is never 0; $q(x) = x^2$ is 0 only when $x = 0$; $h(x) = x^2 - 1$ is 0 at $x = 1$ and $x = -1$. The third degree polynomial $p(x) = x^3$ is 0 only at 0, while $q(x) = x^3 - x$ is 0 at -1, 0, and 1. To get a consistent view of what is going on, you need to work with the complex numbers.

We will call constant functions like $f(x) = 5$ for all real x, polynomials of degree 0.

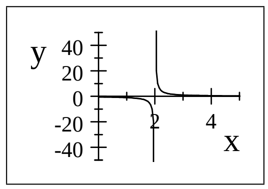
If we have a power of x, the behavior depends on whether the power is even or odd. If the power is even, $\pm x^2$ gives a good idea of the behavior; if the power is odd, $\pm x^3$ gives a good idea. More generally, for a polynomial, the term with the highest power gives a good idea of what happens at $\pm\infty$ since

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = a_n x^n \left(1 + \frac{a_{n-1}}{a_n}\frac{1}{x} + \cdots + \frac{a_0}{a_n}\frac{1}{x^n}\right),$$

and all the terms go to zero at $\pm\infty$ except $a_n x^n \cdot 1$.

8. Rational Functions

A **rational function** is a ratio of polynomials. The domain can be the set of all real numbers where the denominator is not zero. For example, $(x^3 + x)/x$ is a rational function not defined at 0. After division, one gets $x^2 + 1$, so the rational function can be extended "continuously" to take the value 1 at $x = 0$. Another example is $(x^3 - 2)/(x + 5)$. At points where the denominator is 0 and the numerator is not, the graph has a vertical asymptote. That is, the graph approaches a vertical line. An example is $1/(x - 2)$.

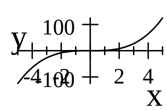


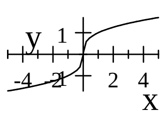
9. Inverse Functions

For certain functions $y = f(x)$, it is also true that x is a function of y, i.e., $x = g(y)$. That is, $x = g(f(x))$ and $y = f(g(y))$, so each function undoes the other. You may have to limit the domains of f and g so that this works. We say that the functions f and g are **inverses** of each other. We can go from the graph of f to the graph of g, by rotating about the line $y = x$.

EXAMPLES:

$y = x^3$ and $x = y^{1/3}$ work as inverse functions for all real values of x and y. Instead of writing $x = y^{1/3}$, we write $y = x^{1/3}$ for the inverse function. Here is a plot of $y = x^3$ and its inverse.





The function $y = x^2$ works for all real values of x, but if we want an inverse, we must either take $x \geq 0$, or $x \leq 0$. For the first case, changing the names of the variables, $y = \sqrt{x}$ is the inverse function,and for the second, $y = -\sqrt{x}$ is the inverse function.

Simple examples (without changing the names) are $y = f(x) = x + 1$ and $x = g(y) = y - 1$; $f(x) = 5x$, and $g(y) = y/5$. These work for all real values. An example which works for all nonzero values is $f(x) = 1/x$, and $g(y) = 1/y$.