1. A Tutoring Room is Open

7–9 p.m, Monday, Tuesday, Wednesday, Thursday, Room 140 Lincoln Hall.

2. Homework 21 due Tuesday, November 7 at 9 a.m.

Section 5.6: #2, 6, 10, 14, 16, 22, 24, 26, 30, 36.

You may have to enter without evaluating. For example, write $(2^3)/3 - 1$ instead of

5/3.

3. Homework 22 due Thursday, November 9 at 9 a.m.

Section 5.7: #4, 10, 14, 38, 40, 44, 48, 52, 54, 56, 58, 60.

You may have to enter without evaluating. For example, write $(2^3)/3 - 1$ instead of 5/3.

4. Homework 23 due Tuesday, November 14 at 9 a.m.

Section 5.8: #2, 4, 8, 14, 18, 20, 24, 26, 28, 36.

5. Homework 24 due Thursday, November 16 at 9 a.m.

Section 5.9: #2, 4, 6. The notation T_n denotes the trapezoidal approximation to the integral. The subscritpt n means the the interval $[a, b]$ is divided into n intervals, so $\Delta x = (b - a)/n$.

Section 6.1: #16, 18, 20, 30, 38.

Section 6.2: #2, 4, 6.

6. Written problem for this week

Consider the region between the curves $y = x - 2$ and $y = 4 - x^2$. Set up and evaluate the integral or integrals for the area between the two curves in two ways, first integrate with respect to x, then integrate with respect to y. Note, this is a sample problem.

7. Written problem for NEXT week

Let $R(b)$ be the region between the x-axis and the curve $y = 1/x$ for $1 \leq x \leq b$.

a) What is the area $A(b)$ of the region $R(b)$?

b) If you rotate the region $R(b)$ about the x-axis, what is the resulting volume $V(b)$?

c) What is the limit of the area $A(b)$ as $b \to +\infty$?

d) What is the limit of the volume $V(b)$ as $b \to +\infty$?

8. Volumes by the method of Cross Section

Suppose you have an onion that lies exactly on the interval $[a, b]$ on the x-axis. This means that if you construct planes perpendicular to the x-axis at a and b, the onion lies between the two planes and touches the planes. Suppose that for each x in $[a, b]$, $A(x)$ is the area of the cross section of the onion cut by the plane perpendicular to the x-axis through the point x. If $A(x)$ is a continuous function of x, then the volume of the onion is $\int_a^b A(x)dx$. To see this, we take $\Delta x > 0$, and look at the Δx partition of $[a, b]$. For each interval $[x_{i-1}, x_i]$, we let V_i be the volume of the part of the onion cut by the planes perpendicular to the x-axis through the points x_{i-1} and x_i. If m_i and M_i are the minimum and maximum values of the area function on the interval $[x_{i-1}, x_i]$, we see that

$$m_i \cdot \Delta x_i \leq V_i \leq M_i \cdot \Delta x_i.$$

It follows that each upper sum for the function $A(x)$ is too big and each lower sum is too small, so the volume is the integral $\int_a^b A(x)\,dx$.

EXAMPLE. Suppose an onion takes the form of a sphere of radius r with center 0 on the x-axis. Find its volume.

Ans. The arc of the top half of the onion is given implicitly by $x^2 + y^2 = r^2$, and so its volume is is the integral of an even function. It is

$$\int_{-r}^{r} \pi(r^2 - x^2)\,dx = 2\int_0^r \pi(r^2 - x^2)\,dx = 2\pi(r^2x - \frac{1}{3}x^3]_0^r = 2\pi(\frac{2}{3}r^3) = \frac{4}{3}\pi r^3.$$

As it should be, this is the volume of a sphere of radius r. You can work similar problems where the axis is the y-axis. The only difference is that now the area is a function of y and the integral is with respect to y.

Sample Problem. The base of a certain solid is a circle of radius a and center 0 in the x-y-plane. Find the volume of the solid if each cross section perpendicular to the x-axis is a square.

Ans. For each x between $-a$ and a, the base of the cross section has length $2\sqrt{a^2 - x^2}$, so $A(x) = 4(a^2 - x^2)$. This is an even function of x. It follows that the volume of the solid is

$$\int_{-a}^{a} 4(a^2 - x^2)\,dx = 2\int_0^a 4(a^2 - x^2)\,dx = 8(a^2x - \frac{1}{3}x^3]_0^a = 8(a^3 - \frac{1}{3}a^3) = \frac{16}{3}a^3.$$

9. Solids of revolution, Method of Washers

Let $y = f(x)$ be a positive, continuous function of x on the interval $[a, b]$. If we rotate the area between the graph of f and the interval $[a, b]$ on the x-axis around the x-axis, we get a solid. At any x in $[a, b]$, the cross section of this solid is a disk of radius

$y = f(x)$, so the area of the cross section is $A(x) = \pi(f(x))^2 = \pi y^2$. Notice that the use of y here as a function of x. It now follows that the volume of the solid is

$$V = \int_a^b A(x)\,dx = \int_a^b \pi(f(x))^2\,dx = \int_a^b \pi y^2\,dx.$$

If instead of x as the independent variable, we use y, and rotate a function $x = g(y)$ for $c \le y \le d$ about the y-axis, then the volume is

$$V = \int_c^d \pi(g(y))^2\,dy = \int_c^d \pi x^2\,dy.$$

EXAMPLE: If we rotate the area between the curve $y = \sqrt{r^2 - x^2}$ and the x-axis about the x-axis, we get a ball of volume

$$V = \pi \int_{-r}^{r} (r^2 - x^2)\,dx = \pi \left[r^2 x - \frac{1}{3}x^3 \right]_{-r}^{r} = \frac{4}{3}\pi r^3.$$

EXAMPLE: If we rotate the area between the y-axis and the line $x = \frac{r}{h}y$ from $y = 0$ to $y = h$, we get a cone of height h and radius for the base r. The volume of this cone is

$$V = \int_0^h \pi \left(\frac{r}{h}y\right)^2 dy = \pi \frac{r^2}{h^2} \int_0^h y^2\,dy = \frac{\pi r^2 h}{3}.$$