1. A Tutoring Room is Open

7–9 p.m, Monday, Tuesday, Wednesday, Thursday, Room 140 Lincoln Hall.

2. Exam, Friday October 17, 11 a.m.

On material through volumes of rotation (homework for Thursday).

Section 4 (Liu Qi), Section 5 (Liu Qi) Section 6 (Michael Barrus), Section 8 (Scott Weaver) will take the exam in Room 314 Altgeld Hall.

Section 2 (Isaac Goldbring), Section 7 (Isaac Goldbring), Section 9 (Timothy LeSaulnier) will take the exam in Room 100 MSEB (Materials Science Engineering Building, North-West corner of Green and Mathews.) People in these sections **must** go to this room and not Altgeld Hall to take the exam.

Everyone should by now know their discussion section and section instructor. You will need to enter that on your examination. Bring your U of I identity card to show when turning in the exam.

Review Thursday September 16, Rooms 245, 443, 445 Altgeld Hall, 7-9 p.m.

3. Homework 23 due Tuesday, November 14 at 9 a.m.

Section 5.8: #2, 4, 8, 14, 18, 20, 24, 26, 28, 36.

4. Homework 24 due Thursday, November 16 at 9 a.m.

Section 5.9: #2, 4, 6. The notation T_n denotes the trapezoidal approximation to the integral. The subscript n means the the interval $[a, b]$ is divided into n intervals, so $\Delta x = (b - a)/n$.

Section 6.1: #16, 18, 20, 30, 38.

Section 6.2: #2, 4, 6.

5. Written problem for this week

Let $R(b)$ be the region between the x-axis and the curve $y = 1/x$ for $1 \leq x \leq b$.

a) What is the area $A(b)$ of the region $R(b)$?

b) If you rotate the region $R(b)$ about the x-axis, what is the resulting volume $V(b)$?

c) What is the limit of the area $A(b)$ as $b \to +\infty$?

d) What is the limit of the volume $V(b)$ as $b \to +\infty$?

6. Defining $\ln x$ as an integral

We began the discussion of the exponential function and its inverse by defining e. On the other hand, one can begin the discussion of the exponential and logarithm function by setting $\ln x = \int_1^x \frac{1}{t}\,dt$ for $x > 0$. It then follows that $D_x \ln x = \frac{1}{x}$, and the second derivative is $\frac{-1}{x^2}$. This gives the fact that the function $y = \ln x$ is increasing, concave down, negative for values of $x < 1$, and positive for values of $x > 1$. One then shows that this function has the properties of a logarithm as follows. Set $L(x) = \int_1^x \frac{1}{t} dt$ for $t > 0$. Then $D_x L(x) = 1/x$, and $L(1) = 0$. For $a > 0$, $D_x L(ax) = 1/x$. Therefore, $L(ax) = L(x) + C$. When $x = 1$, we get $L(a) = C$, so we have $L(ax) = L(a) + L(x)$. That is, L has the property of a logarithm function.

We can now define e by the formula $\ln e = 1$. Since ln is strictly increasing, it has an inverse. Let exp denote the inverse function. For any rational number r, we have

$$e^r = \exp\left(\ln\left(e^r\right)\right) = \exp\left(r \ln e\right) = \exp(r),$$

so $\exp x$ and e^x agree for all rational values of x. We extend to all real numbers.

It follows that $D_x\, e^x$ exists and equals e^x for all real x. To see the latter fact, we note that for $y = e^x$, $\ln y = x$, so $(1/y)(dy/dx) = 1$, that is, $dy/dx = y = e^x$.

It is in this way that we can define the exponential e^x for all real values of x and $\ln x$ for all positive values of x.

7. Other bases for logarithms

For logarithms, we use the fact that for $a > 0$, $b > 0$, and $x > 0$,

$$x = a^{\log_a x} = \left(a^{\log_a b}\right)^{\log_b x}$$

so taking $\log_a$ we have

$$\log_a x = \left(\log_a b\right)\left(\log_b x\right).$$

Since

$$1 = \log_a a = \left(\log_a b\right)\left(\log_b a\right), \qquad \log_a b = \frac{1}{\log_b a}.$$

It now follows that

$$\log_a x = \left(\log_a e\right)\left(\ln x\right) = \frac{\ln x}{\ln a}.$$

Therefore, in working with $\log_a x$, we may work instead with the constant $\frac{1}{\ln a}$ times $\ln x$. Notice again that $\ln e = 1$. We now have

$$D_x \log_a x = D_x \frac{\ln x}{\ln a} = \frac{1}{x \cdot \ln a}, \quad \text{and} \quad \int \frac{1}{x \cdot \ln a} dx = \log_a |x| + C.$$

Sample Problem: Evaluate $D_x \log_{10} x$. **Ans:** $D_x \frac{\ln x}{\ln 10} = \frac{1}{x \cdot \ln 10}$.

For exponentials, we have for $a > 0$,

$$a^x = \left(e^{\ln a}\right)^x = e^{x \ln a}.$$

In fact, if we started with the integral $\int_0^x \frac{1}{t} dt$ as the definition of $\ln x$ and then define e^x as the inverse of $\ln x$, we would define a^x as $e^{x \ln a}$.

For the derivative,

$$D_x a^x = D_x e^{x \ln a} = \ln a \cdot e^{x \ln a} = \ln a \cdot a^x.$$

Notice again that $\ln e = 1$.

Sample Problem: Evaluate $\int \frac{dx}{x \cdot \log_2 x}$. **Ans:**

$$\int \frac{dx}{x \cdot \log_2 x} = \int \frac{dx}{x \cdot \frac{\ln x}{\ln 2}} = \ln 2 \int \frac{dx}{x \cdot \ln x} = \ln 2 \cdot \ln|\ln x| + C.$$

The bottom line is, instead of working with a^x and $\log_a x$, we work with $e^{x \cdot \ln a}$ and $\frac{\ln x}{\ln a}$.

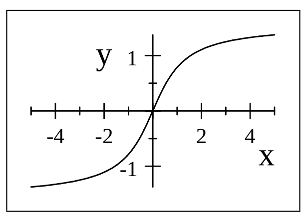
Here is a proof that for $x > 0$ and any number r, $D_x x^r = r x^{r-1}$.

$$D_x x^r = D_x \left(e^{r \ln x}\right) = e^{r \ln x} \cdot D_x (r \ln x) = e^{r \ln x} \cdot \frac{r}{x} = x^r \cdot \frac{r}{x} = r x^{r-1}.$$

8. ARCTAN

The $\arctan x = \tan^{-1} x$ is the angle whose tangent is x. The domain is the whole real line, but the values taken are between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. I prefer to write $\arctan x$ instead of $\tan^{-1} x$. Here is a partial plot:

$\arctan x$



To find the derivative, we set $y = \arctan x$, so $\tan y = x$. Differentiating both sides with respect to x (and using the chain rule), we get $\sec^2 y \cdot \frac{dy}{dx} = 1$. That is,

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}.$$

In general, we have $D_x \arctan u(x) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$.

EXAMPLE: $D_x \arctan x^3 = \frac{1}{1+x^6} \cdot 3x^2$.

EXAMPLE: $\int \frac{dx}{1+x^2} = \arctan x + C$.

Sample Problem: Evaluate $\int \frac{dx}{4+x^2}$. **Ans:**

$$\int \frac{dx}{4 + x^2} = \frac{1}{4} \int \frac{dx}{1 + \left(\frac{x}{2}\right)^2} = \frac{1}{2} \arctan \frac{x}{2} + C.$$