1. A Tutoring Room is Open

7–9 p.m, Monday, Tuesday, Wednesday, Thursday, Room 140 Lincoln Hall.

2. Exam, Friday October 17, 11 a.m.

On material through volumes of rotation (homework for Thursday).

Section 4 (Liu Qi), Section 5 (Liu Qi) Section 6 (Michael Barrus), Section 8 (Scott Weaver) will take the exam in Room 314 Altgeld Hall.

Section 2 (Isaac Goldbring), Section 7 (Isaac Goldbring), Section 9 (Timothy LeSaulnier) will take the exam in Room 100 MSEB (Materials Science Engineering Building, North-West corner of Green and Mathews.) People in these sections **must** go to this room and not Altgeld Hall to take the exam.

Everyone should by now know their discussion section and section instructor. You will need to enter that on your examination. Bring your U of I identity card to show when turning in the exam.

Review Thursday September 16, Rooms 245, 443, 445 Altgeld Hall, 7-9 p.m.

3. Homework 24 due Thursday, November 16 at 9 a.m.

Section 5.9: #2, 4, 6. The notation T_n denotes the trapezoidal approximation to the integral. The subscript n means the the interval $[a, b]$ is divided into n intervals, so $\Delta x = (b - a)/n$.

Section 6.1: #16, 18, 20, 30, 38.

Section 6.2: #2, 4, 6.

4. Homework 25 due Tuesday, November 28 at 9 a.m.

Section 6.2: #8, 14, 16, 18, 22, 24, 42.

Section 6.5: #2, 10, 12.

5. Homework 26 due Thursday, November 30 at 9 a.m.

Section 6.5: #18, 20, 26, 28.

Section 6.6: #6, 8, 10, 12.

6. Written problem for after Thanksgiving

Page 417, #46.

7. Written problem for this week

Let $R(b)$ be the region between the x-axis and the curve $y = 1/x$ for $1 \leq x \leq b$.

a) What is the area $A(b)$ of the region $R(b)$?

Ans: $\int_1^b (1/x)dx = \ln b$

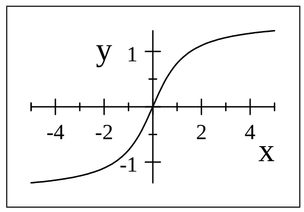
b) If you rotate the region $R(b)$ about the x-axis, what is the resulting volume $V(b)$?

Ans: $\pi \int_1^b (1/x^2)dx = \pi \left[-1/x\right]_1^b = \pi(1 - 1/b)$.
c) What is the limit of the area $A(b)$ as $b \to +\infty$? Ans: $+\infty$.
d) What is the limit of the volume $V(b)$ as $b \to +\infty$? Ans: π.

8. ARCTAN

The $\arctan x = \tan^{-1} x$ is the angle whose tangent is x. The domain is the whole real line, but the values taken are between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. I prefer to write $\arctan x$ instead of $\tan^{-1} x$. Here is a partial plot:

$\arctan x$



To find the derivative, we set $y = \arctan x$, so $\tan y = x$. Differentiating both sides with respect to x (and using the chain rule), we get $\sec^2 y \cdot \frac{dy}{dx} = 1$. That is,

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}.$$

In general, we have $D_x \arctan u(x) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$.

EXAMPLE: $D_x \arctan x^3 = \frac{1}{1+x^6} \cdot 3x^2$.

EXAMPLE: $\int \frac{dx}{1+x^2} = \arctan x + C$.

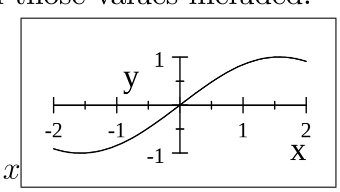
Sample Problem: Evaluate $\int \frac{dx}{4+x^2}$. **Ans:**

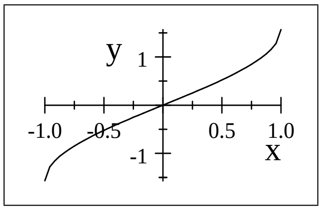
$$\int \frac{dx}{4 + x^2} = \frac{1}{4} \int \frac{dx}{1 + \left(\frac{x}{2}\right)^2} = \frac{1}{2} \arctan \frac{x}{2} + C.$$

9. ARCSINE

Consider the function $\arcsin x$ also written $\sin^{-1} x$. This means the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ necessary to obtain $\sin x$. The function arcsin is defined for all real numbers between -1 and 1, with those numbers included. It takes its values between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ with those values included.

$\sin x$



arcsin x 

For example, $\arcsin(-1) = \frac{-\pi}{2}$, $\arcsin\frac{-1}{\sqrt{2}} = \frac{-\pi}{4}$, $\arcsin 0 = 0$, $\arcsin\frac{1}{\sqrt{2}} = \frac{\pi}{4}$, and $\arcsin 1 = \frac{\pi}{2}$. If $y = \arcsin x$, then $x = \sin y$. By implicit differentiation, $1 = \cos y\,\frac{dy}{dx}$, and $\cos y$ is nonnegative for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, so $\cos y = +\sqrt{1-\sin^2 y}$. Therefore, for $-1 < x < 1$ (notice the strict inequality),

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}.$$

This means that on the **open** interval $(-1, 1)$,

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C.$$

The function arccos is similar. That is, if $y = \arccos x$, then $x = \cos y$. We fix x with $-1 \le x \le 1$ and y with $0 \le y \le \pi$.

$$\frac{dy}{dx} = \frac{1}{-\sin y} = \frac{1}{-\sqrt{1-\cos^2 y}} = \frac{-1}{\sqrt{1-x^2}}.$$

For integration, stay with arcsin.

EXAMPLES: For $x < 0$, $D_x \arcsin e^x = \frac{e^x}{\sqrt{1-e^{2x}}}$.
Sample Problem: For $-2 < x < 2$, evaluate $\int \frac{1}{\sqrt{4-x^2}}\,dx$. **Ans:**

$$\int \frac{1}{\sqrt{4-x^2}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx = \arcsin\frac{x}{2} + C.$$

10. Hyperbolic functions

You will find it useful later on to work with what are called the hyperbolic functions. In particular

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \text{and} \qquad \cosh x = \frac{e^x + e^{-x}}{2}.$$

It is easy to see that $\sinh x$ is an odd function (i.e., $\sinh(-x) = -\sinh x$) and $\cosh x$ is an even function (i.e., $\cosh(-x) = \cosh x$). Moreover,

$$D_x \sinh x = \cosh x, \quad \text{and} \quad D_x \cosh x = \sinh x.$$

That is, each is the derivative of the other. Moreover, each satisfies the differential equation $f'' - f = 0$. Note that the sine and cosine functions satisfy the equation $f'' + f = 0$. There are a number of reasons to study these functions for applications, and the study of hyperbolas. You will run across these in the study of differential equations and complex variables.

You should know the definitions and the derivatives.

Sample Problem: Evaluate $D_x \sinh(e^x)$. **Ans.** $D_x \sinh(e^x) = e^x \cdot \cosh(e^x)$.