1. A TUTORING ROOM IS OPEN

7–9 p.m, Monday, Tuesday, Wednesday, Thursday, Room 140 Lincoln Hall.

2. HOMEWORK 25 DUE TUESDAY, NOVEMBER 28 AT 9 A.M.

Section 6.2: #8, 14, 16, 18, 22, 24, 42.
Section 6.5: #2, 10, 12.

3. HOMEWORK 26 DUE THURSDAY, NOVEMBER 30 AT 9 A.M.

Section 6.5: #18, 20, 26, 28.
Section 6.6: #6, 8, 10, 12.

4. HOMEWORK 27 DUE TUESDAY, DECEMBER 5 AT 9 A.M.

Section 6.7: #4, 6, 12, 18, 26, 28.
Section 6.8: #10, 22, 32, 36, 38, 42.

5. HOMEWORK 28 DUE THURSDAY, DECEMBER 7 AT 9 A.M.

Section 8.1: #2, 6, 22, 24, 26, 28, 30, 38.

6. WRITTEN PROBLEM FOR THIS WEEK

Page 417, #46.

7. WRITTEN PROBLEM FOR NEXT WEEK

Here is a problem using the arctan function. A rectangular painting is hung on a wall. How high should the painting be so that the top and bottom subtend the maximum angle for the viewer. We may assume that the viewer is a feet from the picture, and the picture is b feet high. We will also assume that the viewer's eyes are at level $y = 0$, and that the bottom of the painting is at y, which may be positive or negative or 0.

8. GRADES ON EXAM 4

≤ 34, F, 8. $[35, 39]$, D-, 2. $[40, 44]$, D, 2. $[45, 49]$, D+, 5. $[50, 54]$, C-, 12. $[55, 59]$, C, 10. $[60, 64]$, C+, 10. $[65, 69]$, B-, 18 $[70, 74]$, B, 18. $[75, 79]$, B+, 20. $[80, 84]$, A-, 17. $[85, 94]$, A, 37. $[95, 100]$, A+, 10.

9. ANSWERS TO EXAM

1 a) Assume f is an odd continuous function; that is, for each $x \geq 0$, $f(-x) = -f(x)$. Given that $\int_0^7 f(x)dx = 3$, what is the value of the integral $\int_{-7}^0 f(x)\, dx$? **Ans:** -3.

1 b) Assume g is an even continuous function; that is, for all $x \geq 0$, $g(-x) = g(x)$. Given that $\int_0^7 g(x)dx = 3$, what is the value of the integral $\int_{-7}^0 g(x)\,dx$? **Ans:** $+3$.

1 c) Evaluate $\int_2^5 e^{3t^2}dt + \int_5^2 e^{3x^2}dx$. **Ans:** 0.

1d) What is the average value of the function $\sec^2 x$ on the interval $[0, \frac{\pi}{4}]$? **Ans:**

$$\frac{1}{\pi/4}\int_0^{\pi/4} \sec^2 x dx = \frac{4}{\pi}\tan\frac{\pi}{4} = \frac{4}{\pi}.$$

2 a) Evaluate $\sum_{i=1}^3 i^5$. **Ans:** $\sum_{i=1}^3 i^5 = 1^5 + 2^5 + 3^5 = 1 + 32 + 243 = 276$.

2 b) Evaluate $\int (\sin 3x^2)^5 \cdot \cos 3x^2 \cdot 6x\ dx$.

Ans: Let $u = \sin 3x^2$, so $du = \cos 3x^2 \cdot 6x\ dx$. Then

$$\int \left(\sin 3x^2\right)^5 \cdot \cos 3x^2 \cdot 6x\ dx = \int u^5 du = \frac{1}{6}u^6 + C = \frac{1}{6}\left(\sin 3x^2\right)^6 + C.$$

2 c) Change the variable **and** the limits of integration by a substitution, and then evaluate: $\int_0^1 \frac{1}{(x^4+1)^2} \cdot x^3\,dx$. Ans: Let $u = x^4 + 1$. Then $du = 4x^3dx$, or $\frac{1}{4}du = x^3dx$. Now

$$\int_0^1 \frac{1}{(x^4+1)^2} \cdot x^3\,dx = \frac{1}{4}\int_1^2 u^{-2}du = \frac{1}{4}\left[\frac{-1}{u}\right]_1^2 = \frac{1}{8}.$$

3) Let $f(x) = \cos x$ on the interval $[0, \pi/2]$. Write out, **but do not add up**, the following sums for $\Delta x = \pi/6$. (Hint: There are three intervals. If you forget how to evaluate $\cos x$, remember the side opposite the $\pi/6$ angle of the appropriate right triangle has half the length of the hypotenuse.)

Table: $\cos 0 = 1$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\cos\frac{\pi}{3} = \frac{1}{2}$, $\cos\frac{\pi}{2} = 0$.

a) The Riemann sum evaluating at the left of each interval. Since f is decreasing, this is also the upper sum $\overline{A}_f(\Delta x)$. **Ans:** $\left(1 + \frac{\sqrt{3}}{2} + \frac{1}{2}\right) \cdot \frac{\pi}{6}$

b) The lower sum $\underline{A}_f(\Delta x)$. **Ans:** $\left(\frac{\sqrt{3}}{2} + \frac{1}{2} + 0\right) \cdot \frac{\pi}{6}$

c) The trapezoidal approximation using $\Delta x = \pi/6$. Use the method discussed in the lecture. **Ans:** $\left(\frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{0}{2}\right) \cdot \frac{\pi}{6}$

d) Given a continuous function $f(x)$ on an interval $[a, b]$ and $\Delta x > 0$, we have written M_i for the maximum value of $f(x)$ in the i^{th} interval and m_i for the minimum value of $f(x)$ in the i^{th} interval. We then set $E_f(\Delta x) = \max_i(M_i - m_i)$. State the basic fact about $E_f(\Delta x)$ that is needed for integration. **Ans:** $\lim_{\Delta x \to 0} E_f(\Delta x) = 0$.

4 a) Let $F(x) = \int_2^x e^{t^2+3} dt$. What is the derivative $F'(x)$? **Ans:** e^{x^2+3}.

4 b) What is the value $F(2)$? **Ans:** 0.

4 c) Write a formula for the antiderivative of e^{x^2+3} that takes the value 0 at $x = 2$. (Hint: look at Parts a and b.) **Ans:** $\int_2^x e^{t^2+3} dt$.

5) Suppose the mass density of a 3 centimeter long rod with the left end at 0 is given by $\rho(x) = x^3$ grams/cm. Find the mass of the rod. **Ans:** $\int_0^3 x^3 dx = \left[\frac{1}{4}x^4\right]_0^3 = \frac{81}{4}$.

6) Let A be the area of the region to the right of the y-axis bounded by the curves $y = x^3$ and $y = \sqrt{x}$. Note that for $0 \le x \le 1$, $x^3 \le \sqrt{x}$.

a) Find the area A integrating with respect to x. Show all your work. **Ans:**

$$\int_0^1 \left(\sqrt{x} - x^3\right) dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{4}x^4\right]_0^1 = \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}.$$

b) Find the area A integrating with respect to y. Show all your work. **Ans:**

$$\int_0^1 \left(y^{1/3} - y^2\right) dy = \left[\frac{3}{4}y^{4/3} - \frac{1}{3}y^3\right]_0^1 = \frac{3}{4} - \frac{1}{3} = \frac{9-4}{12} = \frac{5}{12}.$$

7) Let R be the region to the right of the y-axis bounded by the x-axis, the curve $y = x^3$, and the vertical line $x = 1$.

a) Find the volume obtained by rotating the region R about the x-axis.

Ans: $\pi \int_0^1 x^6 dx = \pi \left[\frac{1}{7}x^7\right]_0^1 = \frac{\pi}{7}$.

b) The base of a certain solid is the region R. Find the volume of the solid if each cross section perpendicular to the x-axis is a square. **Ans:** $\int_0^1 x^6 dx = \left[\frac{1}{7}x^7\right]_0^1 = \frac{1}{7}$.