Math 241 Quiz 4 Version D

1 (3pts) Find f_x, f_y and f_{xx} of $f(x, y) = y\cos(xy) + x^3$.

First we compute that

$$f_x = -y^2 \sin(xy) + 3x^2 \tag{1}$$

We then have that

$$f_y = \cos(xy) - xy\sin(xy) \tag{2}$$

and finally using (1) again we have that

$$f_{xx} = -y^3 \cos(xy) + 6x \tag{3}$$

2. (4 pts) Find equations of tangent plane and normal line to the surface $z = x^2 + xy^3$ at the point, where $x = -2$, $y = 1$

Look at the surface

$$f(x, y, z) = x^2 + xy^3 - z = 0 \tag{4}$$

If we find the gradient at f we then have that

$$\bigtriangledown f(x, y, z) = \langle 2x + y^3, 3y^2x, -1\rangle \tag{5}$$

Evaluating $x = -2$ and $y = 1$ and solving for z we find that $f(-2, 1, z) = 4 - 2 - z = 0$ or $z = 2$. This means that

$$\bigtriangledown f(-2, 1, 2) = \langle -3, -6, -1\rangle \tag{6}$$

So the tangent plane to the surface is

$$-3(x+2) - 6(y-1) - (z-2) = 0 \tag{7}$$

or if we multiply by -1 we get that

$$3(x+2) + 6(y-1) + (z-2) = 0 \tag{8}$$

is the tangent plane to the surface.

The normal line to the surface is

$$q(t) = \langle -2 - 3t, 1 - 6t, 2 - t \rangle \tag{9}$$

3. (3 pts) Let $g(u, v) = f(x, y)$ for $x = u - 2v$ and $y = u^2 v$. Find g_u and g_v.

We know that

$$g_u = f_x \frac{\partial x}{\partial u} + f_y \frac{\partial y}{\partial u} = f_x + f_y(2uv) \tag{10}$$

also

$$g_v = f_x \frac{\partial x}{\partial v} + f_y \frac{\partial y}{\partial v} = f_x(-2) + f_y(u^2) \tag{11}$$

This can be easily summarized as

$$\begin{pmatrix} g_u \\ g_v \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{pmatrix} \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} 1 & 2uv \\ -2 & u^2 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{12}$$